NORTEK MANUALS

Principles of Operation WAVES

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1 Introduction

This manual is designed to give an overview of the principles of operation when using a Nortek instrument to measure waves. Details about different processing methods and wave parameters can be found in the chapters that follow. Most information found here is general and relevant for all different types of current meters and profilers. For instrument specific details related to operation and configuration, please refer to the deployment software. For information about current measurements, please refer to the Principles of Operation - Currents.

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2 Ocean waves background

Waves are a captivating and powerful natural phenomena. The creation of waves begins with any force that disrupts water. These forces can be gravitational effects from the sun and the moon, wakes from vessels or swimmers, or even underwater disturbances from earthquakes and volcanoes. Once waves are formed they travel across all bodies of water and carry massive amounts of energy. The impact of wave formation is extensive and effects a wide variety of applications including commercial shipping, offshore energy development, coastal protection, and navigation.

To better understand characteristics of waves it is helpful to consider the waves in basic mathematical terms. To do this, scientists and researchers use simple sinusoidal waves to describe the surface because they can be expressed as a single sine or cosine function (Figure 1). By doing this, waves can be easily studied because the characteristics of these functions are uniform and constant.

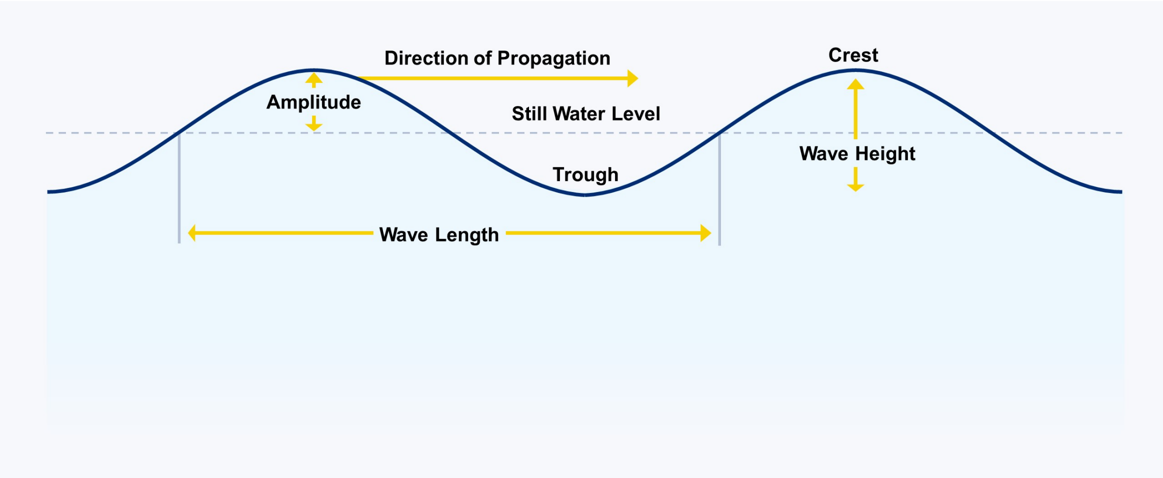

Figure 1: Wave characteristics of a basic wave

- Still Water Level - The level of the sea surface if there were no waves present.
- Crest - The highest point of a wave
- Trough - The lowest point of a wave.
- Wave Height - The vertical distance between the crest and the trough.
- Wavelength - The length of one complete cycle of a wave. This is also the distance between two successive peaks.
- Amplitude - The distance to the crest from the still water level or one half of the wave height.
- Direction of Propagation - The direction in which a wave is propagating through space.
- Wave Period - The time interval between two successive peaks. This is also how long the wave takes to travel one wavelength in distance
- Wave Frequency - The number of peaks that pass a fixed point per second.

Note that the wave frequency is the inverse of the wave period, so that:

$$Frequency = \frac{1}{Period} \tag{1}$$

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All bodies of water experience waves and the forces behind the waves vary. Storm surges can be created by strong wind and pressure that can induce long waves in deep water that gain strength as they propagate towards the shore. Underwater disturbances such as earthquakes, or volcanic eruptions can create increasingly long waves that form into Tsunamis. The gravitational forcing of the sun and moon can also create tidal waves. If you look at the distribution of energy for waves, you can see considerable variability ranging from 12 hours to 0.5 seconds. A significant contribution of this energy is found in the band from 0.5 to 30 seconds and is commonly referred to as wind waves (Figure 2).

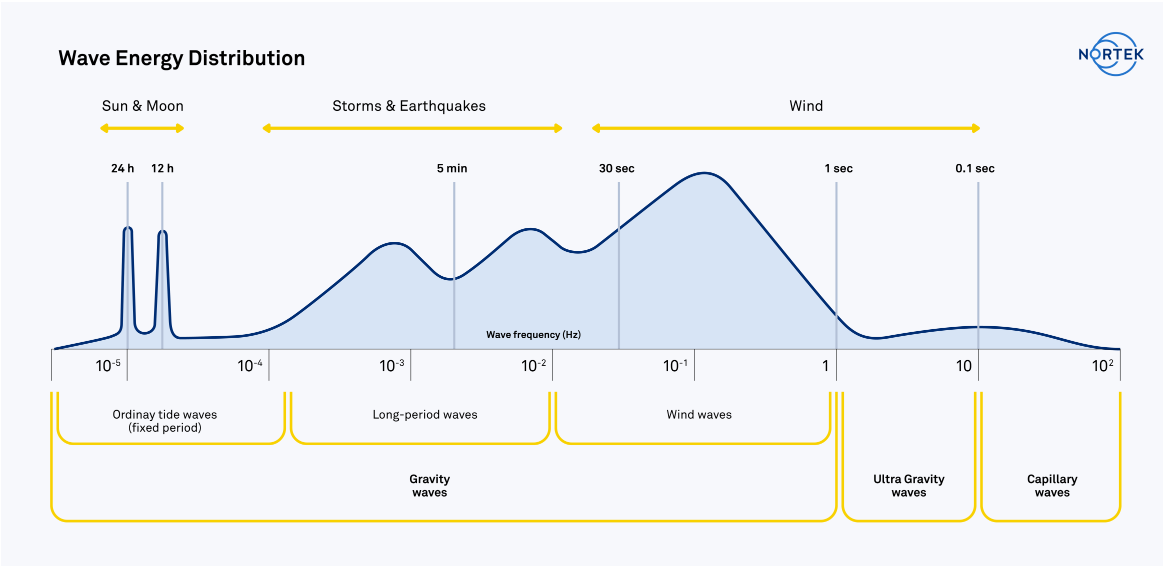

Figure 2: Types of surface waves, showing relationship between the forces that cause them and their frequency or period. The blue shaded area shows the relative energy of each wave frequency. The higher the shaded area, the more energy that frequency has.

Waves generated by the wind are the most common type of ocean waves and are the primary focus for many engineers and scientists. When wind blows across the surface of the water, waves are created by the energy transferred to the water from the friction between the wind and water surface. The energy flows through the water causing it to move in a circular motion, but the water itself does not actually advance with the wave. This is commonly observed when viewing a buoy floating at the surface come in contact with a wave in open water. The object may bob up and down and even appear to have a circular pattern, but it returns very close to its original position.

Wind waves have a variability that makes characterizing waves non-trivial. Waves start out as both small in height and short in length, created by local winds and grow as a function of wind strength, duration of wind, and distance. As a result, the wave environment at a particular location may be composed of a combination of local wind waves from a sea breeze and long waves (swell) generated by storm events hundreds or thousands of kilometers away. When measuring waves we therefore need to take into account that the local sea state is composed of waves with different amplitudes, periods, and directions. Understanding this is the first step towards making accurate wave measurements.

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3 Subsurface wave properties

Waves on the ocean surface are visible to us all, however, less obvious are the subsurface dynamics generated by these waves. As explained in the Ocean Waves Background 5 chapter, expressing waves in simple mathematical terms as sine and cosine functions, is beneficial when analyzing their characteristics and behavior. However, when observing a body of water it is clear that waves are not in fact uniform and have quite a bit of variability. At any given time the sea surface is composed of different types of waves the surface is in general irregular in both time and space. As a manufacturer of Doppler instruments, Nortek's wave measurement solutions approach the problem from below the surface. A major advantage of such subsurface measurements is that the instrumentation is located safely below the surface where the risk of loss by vessel collision, vandalism, or theft is reduced.

3.1 Orbital Velocities

Beneath the surface, waves generate an orbital motion as they pass a point (Figure 3). When a wave propagates past a point it creates local currents below the surface in a clockwise direction. These currents are special in the sense that they are changing direction, so that the water particles below the crest of the wave moves in the direction of propagation, while the water below the trough moves in the opposite direction. This cyclical motion constructs a circular path in deep water and is often referred to as a wave's orbital velocity.

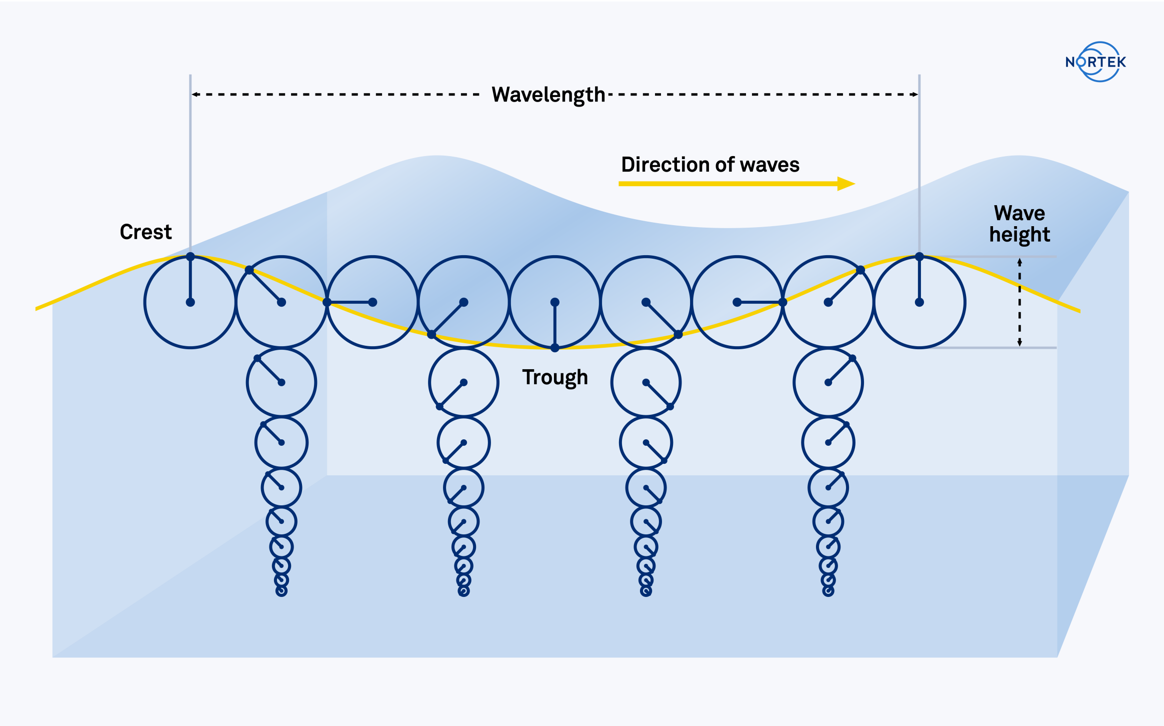

Figure 3: Description of the wave orbital velocities beneath a wave as it propagates in deep water. Note that the orbital velocity attenuates with depth.

The ability to measure the orbital dynamics from below allows us to interpret the waves on the surface by use of linear wave theory, and estimate many of the wave parameters that are commonly used to describe a sea state. An important detail to understand about orbital velocities is that they attenuate exponentially with increased depth and shorter wavelength. In other words, particle velocities and their orbital dimensions decrease with increasing distance below the surface and wave energy will only propagate to a certain depth and consequentially the energy cannot be seen or measured below this depth. This means that short waves in deep water do not have an orbital velocity

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signal that penetrates to the bottom, since higher frequency waves attenuate more quickly with depth. Thus there exists a tradeoff between the depth of the measurement location and the ability to measure the higher frequency waves. This is also true for the dynamic pressure (discussed in the next section), since it is largely dependent on the presence of orbital velocities.

A wave is said to be a deep water wave when the total water depth (L) is greater than half the wavelength (>L/2). The orbital motion for deep water waves is circular and of diminishing diameter with depth. Conversely, a wave is considered a shallow water wave when the total water depth is less than 1/20 of the wavelength (<L/20). The orbital motion for shallow water waves is elliptical at the surface getting progressively flatter with depth. A the bottom, all motion is strictly back and forth. When the wave is at a depth less than L/2 but greater than L/20 it is a transitional wave.

3.2 Dynamic Pressure

Another important property when measuring waves is the dynamic pressure. It is largely dependent on the presence of orbital velocities and this means that similarly to the orbital velocities the dynamic pressure signal from waves experiences attenuation as a function of depth and wavelength (Figure 4). The dynamic pressure is at maximum under the wave crest. The rate of decrease with depth is well understood and modeled by linear wave theory. This allows us to measure the pressure near the bottom, and to rescale the measurement to obtain the wave elevation spectrum at the surface by use of transfer functions.

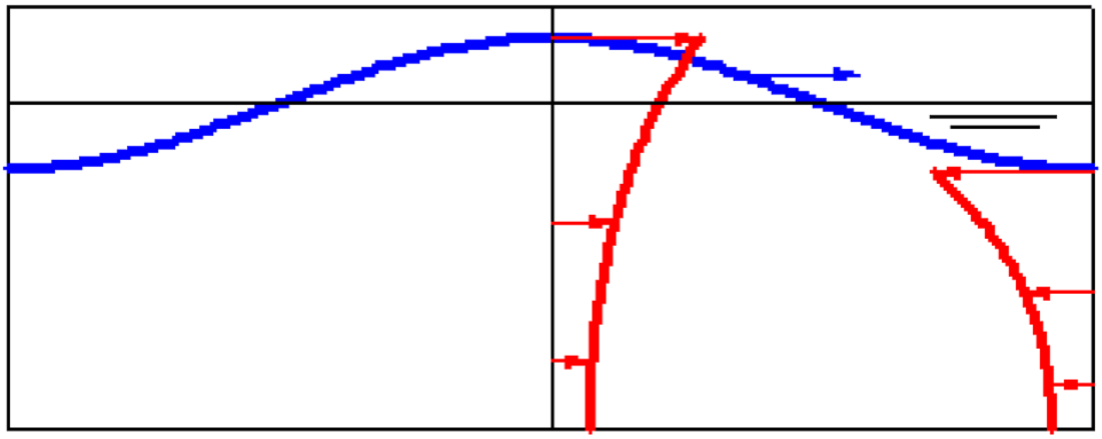

Figure 4: A wave moving in the direction of the blue arrow. The pressure profile is shown in red. Pressure and velocity (red arrows) under the crest are in phase with eachother.

4 Statistical approach

Since the sea surface is composed of different types of waves and in general is irregular in both time and space, the sea state is characterized by statistics (wave height, period, direction etc.). A couple of assumptions are necessary when trying to approximate the wave field; the first is that the wave field can be described as a summation of many different sine waves with different frequencies, amplitudes and directions. This makes it possible to use Fourier analysis to reduce a time series of waves to a certain number of sine waves. Fourier analysis is a mathematical method used to break down and transform a periodic function into a set of simpler functions (e.g. sine and cosine) thereby providing a simpler, general solution. The second assumption is that the wave field is statistically stable, meaning that the same statistical result would be obtained if the same wave measurements were made just a moment later (a so-called short-term description).

4.1 Time series analysis

The most intuitive method for estimating wave parameters is to evaluate the time series of sea surface displacement from a single measurement point. A time series is a way of studying a sequence of repeated measurements over a specified period of time. By analyzing the time series of surface measurements, it allows us to extract useful statistical information about the waves and better understand their characteristics and behavior. The resulting time-series analysis determines how far the water surface extends above and below the mean water level. Individual waves can then be determined by where the trace crosses the mean level and defines the start and stop of the individual waves; this is commonly known as the zero-crossing method. See an example in Figure 5 of a time series of the surface where the zero-crossing method is used to determine individual wave heights (H) and wave periods (L).

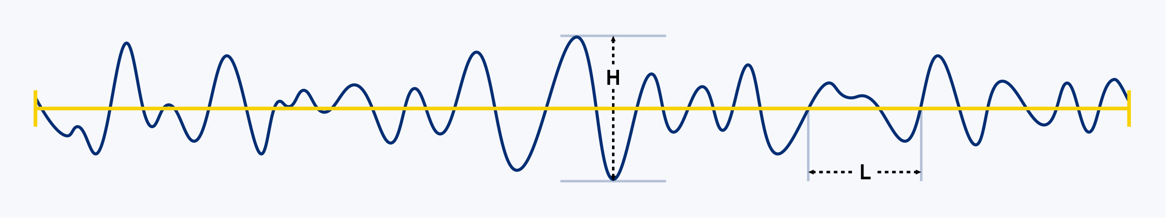

Figure 5: Example of a time series of the surface. Here the zero-crossing technique is used to determine individual wave heights (H), and wave periods (L).

The individual waves in the record can be characterized by period (defined by where it crosses the mean water level) and height (defined by the distance from trough to crest between crossings). The result of this exercise is a wave record composed of many waves with a variety of heights and periods.

If these waves are ranked by their height and/or period, then the resulting rankings can be used to calculate common estimates of height and period. Two of the more common estimates are Significant Wave Height (Hs) and Mean Period (Tz). The significant wave height, sometimes referred to as H3, is the mean height of the highest one-third of all waves in the record's ranking. This parameter was created by oceanographer Walter Munk during World War II, and is the estimation of wave heights that would be observed from a fixed point at sea. Wave heights are directly connected to wave energy and therefore, in many applications larger waves are more "significant" than smaller waves. It is important to note that the significant wave height is a statistical representation of the waves during the sampling period and does not represent any individual wave. In other words, the waves during this period will be both higher and lower than the determined significant wave height.

The mean period is the mean of all the periods of the record's ranking. Other parameters that may be estimated from the record's ranking are maximum wave height (Hmax), which is simply the largest

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measured wave in the record, and the mean of the largest 10% of all waves in the record (H10). The latter two parameters are commonly used for coastal design and assessment and are only possible when we have a direct measure of the surface displacement. Indirect measurements of waves cannot produce these parameters.

4.2 Spectral analysis

Time-series analysis may seem like the right way to approach wave measurements, but two common restrictions keep many such analysis from succeeding. The first restriction is that time-series analysis can be a little daunting; the second is that many wave-measuring devices do not have the technology to directly measure surface displacement and, therefore, cannot provide the data needed for time-series analysis. Instead, these instruments measure a wave-related property such as pressure or velocity and infer the sea state from the spectra of the time series. A different approach is the spectral analysis, made possible by application of Fourier transforms. A given trace of the waves can be analyzed by using Fast Fourier Transform (FFT) to produce energy density spectra (see Figure 6). The spectrum shows how the energy density is related to different frequencies. Having obtained a spectrum, the frequency domain wave parameters may be found. Both the ease of interpretation and large number of non-direct measuring instruments has left spectral analysis as the primary method for processing wave results. It provides an enriched collection of wave parameters and also permits directional wave analysis.

Wave parameters resulting from spectral analysis include Peak Period, Peak Wave Direction and the spectral analogs of mean period and significant wave height, denoted as Tm02 and Hm0, respectively. The most complete solution is to carry out both a time-series analysis and spectral analysis.

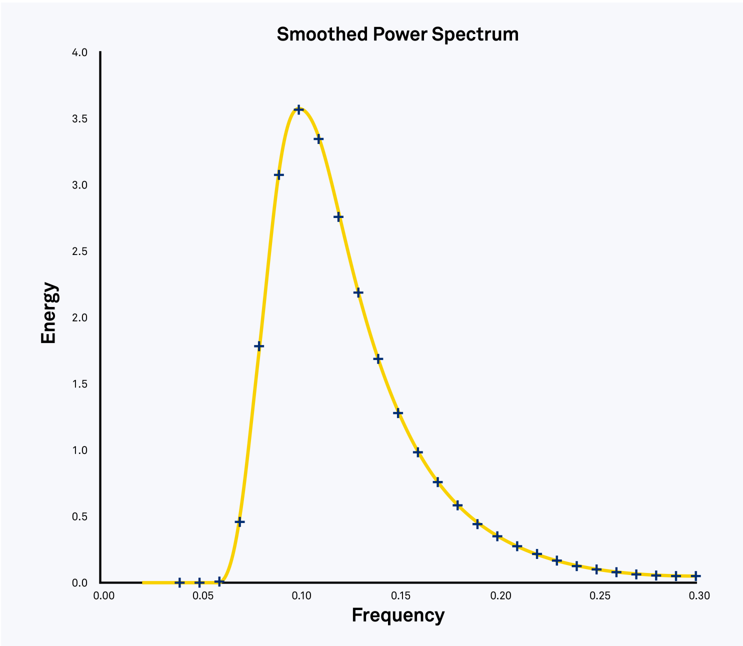

Figure 6: Energy density spectra for a time series. Energy is shown in arbitrary units. Note the shape of the curve and the location, along the frequency axis, of where the peak is.

5 Wave measurement solutions and processing methods

When measuring waves, the instrument collects bursts type of data with different parameters such as pressure, velocity and distance to surface. The resulting time series of the raw measurements is not particularly useful from a practical standpoint, and therefore needs to be processed to yield parameters that can broadly, yet accurately, characterize the sea state. The most common wave parameters describe height, period and direction of the wave field, and are single values representative of the burst time series. The processed wave data will be time series of wave parameters, one number per wave burst. The wave processing is either done internally in the instrument to output live data (only relevant for certain instrument types) or in post-processing. We distinguish between directional (direction) and non-directional (height and period) wave parameters.

Depending on the instrument type, you are able to gather different type of information within the wave burst. The information you have dictates how you can process the data. In the following chapter the different wave processing methods applicable for wave data collection with Nortek instruments are described. Each of the following solutions represents a fundamental improvement over the previous technique, reflecting the evolution over time of the way in which subsurface instrumentation measures waves. The improvement gives a gradually extended the high-frequency range as we try to best cover the 0.5–30-second wave band, also from deeper deployed instrumentation.

Before starting your wave data collection, you need to have certain knowledge about the wave conditions at your deployment site, in order to capture as much of the wave spectrum as possible. As discussed in the Subsurface wave properties 7 chapter, the signal from surface waves attenuates with depth, and it is therefore more difficult to measure the smallest waves the deeper your instrument is deployed. The limitations for the different measurement solutions are also discussed in the following sections.

5.1 PUV Method

This method was perhaps the first approach used for measuring directional and non-directional wave properties from below the surface. It dates back to the 1970's and because of its modest requirements for instrumentation and processing, it is still in use to this day. The name itself is a description of the method as it is an abbreviation of the three quantities measured: pressure, and the two horizontal components of the wave's orbital velocity, U and V. These measurements are made at the instrument's deployment depth (Figure 7) and because they are co-located at the same point, this is referred to as a triplet measurement. Nortek instruments that are commonly used for wave measurements using the PUV method is the Aquadopp, the Aquadopp Profiler and the Vector.
The PUV method is unique in that it is able to accurately estimate wave height, period, and wave direction without a direct measurement of the surface by the use of inferred estimates. This is possible because both the dynamic pressure and orbital velocities are driven by the surface waves, as described in the Subsurface wave properties 7 chapter. The signals associated with these properties attenuate exponentially with depth. The dynamic pressure measurement provides a means of estimating all of the non-directional wave parameters, while the combined P, U, and V measurements allow for estimating the directional wave parameters. The exact behavior of the attenuation has to do largely with the depth of the water and the wavelength:

1. As we move down in the water column the signal is increasingly attenuated
2. As the wave length decreases (shorter period and higher frequency) the signal again experiences increasing attenuation.

In other words, waves become more difficult to estimate when they are measured from great depths or are short in period. This means we are both depth and frequency limited when measuring waves.

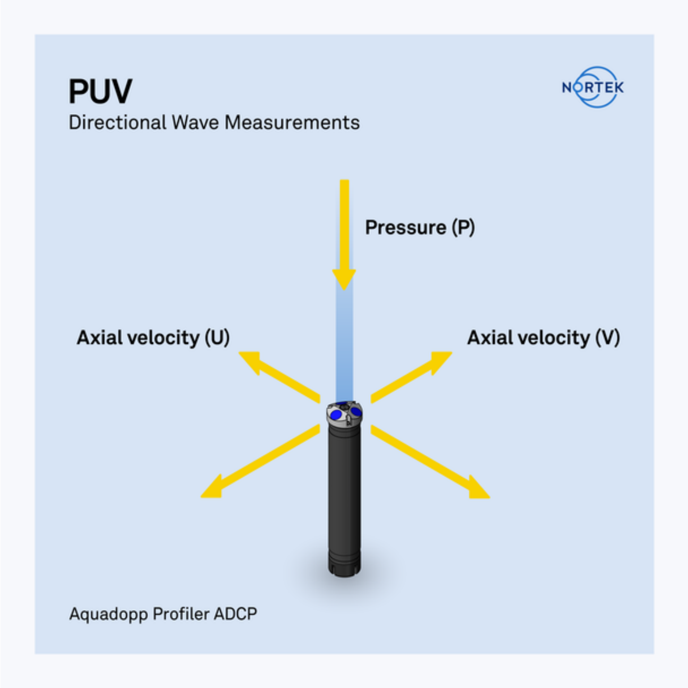

Figure 7: General schematic of a Nortek Aquadopp Profiler ADCP showing orientation of horizontal velocity (U and V) and pressure components (P) that together make up the PUV method.

Understanding the frequency limitations and characteristics of the waves that are to be captured is critical to determining the instrument's depth and ultimately successful data collection. Therefore we aim to achieve a 'Target Depth', which is a deployment depth at which we capture as much of the wave spectrum as possible with the greatest amount of accuracy. The accuracy of the solution improves as the more of the wind wave band (waves with periods of 0.5-30 seconds) is covered. Incomplete or insufficient coverage of the wind wave band can result in underestimation of wave height and missing peaks in the spectrum.

To simplify deployment conditions and optimize data quality, Nortek recommends the following limitations when using the PUV method:

1. Deployment depths that are shallow (less than 10–15 meters)
2. Waves that are long (approximately periods of 4 seconds or longer).

Although these limitations do not encompass the entire wave band, they have been tested to provide consistent data. The limitation of only measuring long waves (swell) is the one that should raise a warning flag for those who are interested in the complete description of the wave environment. For high end users the table below shows the limitations of the **minimum** measurable waves with regards to depth, frequency, and height. This table is meant to be used as a guideline and the conditions at the deployment site greatly influence the necessary deployment depth, wave frequency and height.

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Depth	Peak Period	Hs
20	5.8	0.75
20	6.4	0.60
20	7.2	0.35
20	8.3	0.30
20	10.1	0.20
15	5.0	0.50
15	5.5	0.40
15	6.2	0.25
15	7.1	0.20
15	8.7	0.15
10	4.1	0.40
10	4.5	0.30
10	5.0	0.20
10	5.8	0.15
10	7.1	0.10
5	2.9	0.20
5	3.2	0.15
5	3.6	0.10
5	4.1	0.07
5	5.0	0.05
3	2.3	0.12
3	2.5	0.10
3	2.8	0.05
3	3.2	0.04
3	3.9	0.03

Table 1: Minimum measurable significant wave heights (Hs) for several different depth and deployment scenarios.

Measuring large waves in the Pacific Ocean?
Begin with the assumption that the waves have the potential to be several meters high and a peak period of approximately 10 seconds. After consulting the table the target deployment depth is 15 meters.

Measuring large waves within a sheltered bay?
Based on historical data the peak period is typically around 5 seconds and height not too large. After consulting the table the target deployment depth would be 5-10 meters.

PUV Processing

The process by which we arrive at wave estimates from the energy distribution requires special attention. Since the wave estimates resulting from the PUV processing method are based on the wave energy distribution, and not a direct measure of the free surface, they are considered inferred estimates. Fourier transforms are used to separate the signals into different frequency bands so that it can determine the direction separately for each band. This means that if you have a long-period swell coming from one direction, and a shorter period coming from another, you can tell the direction for each of them separately. The main assumption for standard PUV wave measurements is that waves at a given frequency come from one primary direction.

The processing steps are relatively simple and are composed of the following steps:

1. First perform the transformation on the time series (e.g. Pressure, Velocity) from the time domain to the frequency domain using a standard FFT (Fast Fourier Transform).
2. Calculate the Auto and Cross Spectra for the pressure and two velocities.
3. Apply the transfer functions to the Auto Spectra to arrive at the Power Spectra for the free surface (see Additional Reading)32.
4. Apply quality control to the spectra (Determine a cutoff frequency and extrapolate).
5. Estimate the wave statistics for height and period using moments calculations.
6. Calculate the Fourier arguments which will ultimately be used for the directional estimates

Correction for Background Currents

In case of strong background currents, the measured waves may be affected by a Doppler shift. That is, when currents are directed against the waves, the waves are compressed. When the currents travel in the same direction, the waves are elongated. The resulting spectra will see the peak energy shift slightly to lower or higher frequencies. It is not just the magnitude of the currents that is essential but also the direction. Currents flowing in a direction perpendicular to the wave direction will have no effect on the waves.

The degree to which the Doppler shift modifies the surface waves depends on the current speed relative to the wave propagation speed. This means that slow propagating (short period) waves are the most affected by currents. Measurements that infer the surface waves from either orbital velocity or pressure measurements require special attention regarding background currents. This is because the transfer function used for inferring the surface waves is wavenumber dependent, and it is the wavenumber that is modified by the background currents. The wavenumber solution must take into account the mean current and direction relative to the wave direction. The post-processing method that relies on the wavenumber solution is the PUV method, and it is the one which is most sensitive to the effects of currents. The correction for background currents is done in post-processing software when necessary. Conversely, Acoustic surface tracking (AST) is a direct measure of the surface waves and therefore its response is unaffected by background currents.

5.2 MLM Method

The shortcomings of the PUV method, related to limitations in depth and period, prompted the development of a new technique for measuring waves, the Maximum Likelihood Method (MLM). This new technique involves employing current profilers to measure orbital velocities closer to the surface where the velocities are less attenuated by depth (Figure 8). As a result, the shorter waves could be measured at greater depths.There is an effective doubling of performance; the deployment depth can be doubled or the cutoff period is reduced by half.

This method, however, is not without its shortcomings. More complex processing methods are required since the measurements are no longer co-located (triplet measurements), but are in the formation of an array of measurement cells (Figure 9). The most common array processing method is the Maximum Likelihood Method (MLM), a method that has demonstrated very favorable results. The Maximum Likelihood Method is rarely used on its own with Nortek instruments, and is most-often combined with AST measurements to create the MLMST time-series method.

Figure 8: General schematic of a Nortek AWAC showing orientation of velocity beams used in the Array method.

Array Method Concept

When measuring waves with an ADCP is that the instrument measures currents in the BEAM coordinate system. This means that the orbital velocity measurements are projected collinearly with the beam. This is why when an array method is used in wave processing, the sinusoidal time series of the velocity is slightly out of phase and with a different amplitude than the other beams. The phase difference is a result of the time lag as the wave passes through the "array" of velocity measurement cells below the free surface. The amplitude difference is attributed to how much of the wave's direction is in line with the projection of the orbital velocity in line with the beam. This concept is demonstrated in Figure 9.

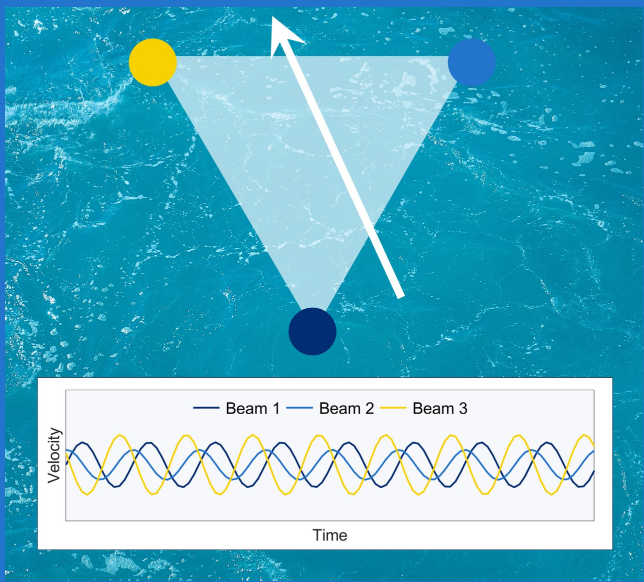

Figure 9: Diagram representing the beam phase difference when the array method is used.

To measure orbital velocities, Nortek instruments use one cell, known as the wave velocity cell. Keep in mind that the wave velocity cell size is often much larger than the cell sizes used in current profiling. During wave processing a suitable wave cell per wave burst is adaptively selected, so it will change depending on the tidal conditions (Equation 2 and 3). In shallow water, the cell size is smaller, whereas in deep water it is larger. The position of the cell is optimized to be as close to the free surface as possible without making contact to allow for strong orbital velocities readings. An acoustic test is performed on the quality of the signal in preceding current measurement data. In clear water with little scatterers, the cell is drawn in closer to the instrument where the SNR is better. It is important to remember that the size and position of the wave velocity cell is not straight forward and changes depending on the instrument.

An important detail about the array solution is that the complete wind wave band (0.5-30 seconds) is still not covered and underestimation is possible if the instrument is deployed in typical coastal depths (e.g. greater than 15 meters). The limitation is a result of the horizontal spacing of the velocity cells that construct the array near the surface. When the spacing is greater than half of the wavelength that is being measured, the solution is no longer valid. If you were to move the cell down along the beam to reduce the distance in the array, the shorter waves would suffer from severe orbital velocity attenuation so nothing is really gained. This is not a severe limitation, but in deeper waters a significant portion of wave information is lost. Array method was a large step forward for subsurface wave measurements, but the complete wind-wave band still needed to be resolved in order to avoid underestimating wave height.

5.3 Acoustic Surface Tracking (AST)

In 2002, the Nortek AWAC was first released with the Acoustic Surface Tracking (AST) option. The vertically oriented transducer in the center of the AWAC could now be used to measure the distance to the sea surface directly by using the simple echosounder principle (Figure 10). The travel time from the instrument to the surface and back allows us to estimate the distance to the surface for each ping. The direct measurement has many advantages; the first, and most profound, is that there is effectively no depth limit for coastal waters and that the largest possible portion of the wind-wave band is covered. The AST measurement also allows for both time-series and spectral analysis. This means design parameters such as H10 and Hmax can be measured directly.

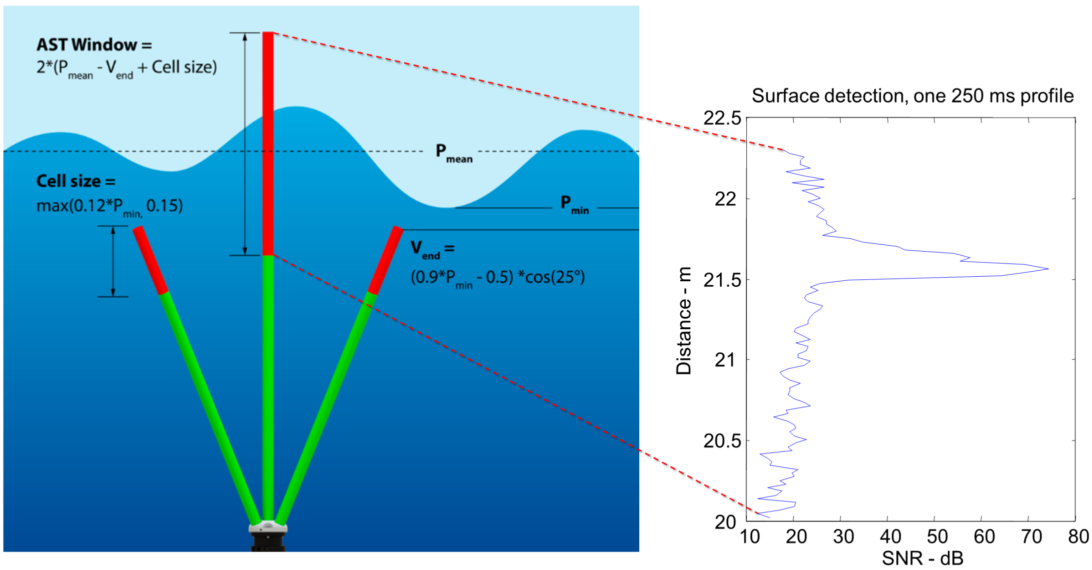

Figure 10: Nortek wave measurement routine shown with an AWAC

To ensure surface detection a relatively large receive window known as an AST window is adaptively configured to provide a strong return at the water-air interface (Figure 9). This is the window where the instrument is looking for the surface. For the AWAC, the AST Window is determined by the pressure readings estimated during the preceding current profile to ensure that the window covers the range of all possible wave heights. During the current profile, the mean and minimum pressure readings are taken to give an indication of where the trough of the passing wave and the mean water level are located. The equations used to determine the AST window is given in Equation 2, 3, and 4.

$$Cell\ size = max(0.12 \times P_{min}, 0.15) \tag{2}$$

$$V_{end} = (0.9 \times P_{min} - 0.5) \times \cos 25° \tag{3}$$

$$AST\ window = 2 \times (P_{min} - V_{end} + Cell\ size) \tag{4}$$

The Signature series determine the AST window slightly differently. The Signature instruments have a relatively large, fixed altimeter window, which is defined in the configuration stage with Altimeter Start and Altimeter End parameters. When estimating the AST value in post-processing, the pressure

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sensor measurement for each altimeter ping is used to center the AST window around the pressure value sufficient for all possible wave heights.

The raw AST data outputted by the instrument is a time series with distances from the instrument to the surface. The approach used by the instrument to detect the surface is relatively simple, and can be broken down into the following sequence of automatic steps:

1. Transmit a relatively short pulse
2. Specify a receive window covering the range of all possible wave heights
3. Discretise the receive window into multiple cells (~5 cm) to achieve high resolution of the surface
4. Apply a match filter over series of cells to locate the maximum peak, which is the surface
5. Use quadratic interpolation of the peak point and its neighbors to precisely estimate surface location, the final resolution of the distance to the surface is 1 mm.

Note that this procedure is carried out internally in the instrument, resulting in a time series with distances to the surface found in the raw wave data.

A cleanup step is iteratively performed on the raw time series prior to processing. The AST time series is linearly detrended, and all detections that deviates with more than 4 standard deviations from the mean of the time series is considered an outlier and called a bad detect. If the cumulative number of false and no detects exceeds 10% of the total number of samples in an ensemble, the ensemble should be considered corrupt and discarded.

The AST is used in two different processing methods, MLMST and SUV. These are described in the following sections.

5.3.1 MLMST Method

The MLMST is a version of the array method, adapted for surface tracking measurements instead of pressure measurements. Wave orbital velocity measurements are still made close to the surface like in the MLM solution, but instead of the dynamic pressure, the AST option is utilized to estimate the nondirectional spectrum. It is important to note that array solutions in wave processing is only an adequate solution for a bottom-mounted instrument that is not rotating. There are still limitations to this method, directional estimates are limited by the horizontal separation of the wave measurement cells and AST, and the nondirectional estimates are limited by the AST footprint (see the Limitations 20 chapter for more details).

5.3.2 SUV Method

In 2005, Nortek patented a new method for processing wave data from current profilers with AST called the SUV method. Today, this method can be used with both AWACs and Signatures. The solution represents a hybrid of the PUV and AST measurements. Wave orbital velocity measurements are still made close to the surface, like in the array solution, but instead of using an array, the velocities are converted to collocated velocity components of U and V. The other difference is that pressure is no longer used, and the AST is used in its place.

The result is a solution that permits the AWAC and any Nortek ADCP with a vertical beam (e.g. most of the Signature ADPCs) to be mounted on a subsurface buoy. This means that when wave measurements are desired in waters where the total depth is too deep for the instrument to be mounted on the seabed and use the array solution, the instrument may be placed on a subsurface buoy and positioned closer to the surface (e.g. 30 meters below the surface). In contrast to the MLMST method, the SUV method accounts for and corrects for the motion of the buoy, making these measurements possible. The instrument measures directional wave characteristics as if it was mounted on a seabed at 30 meters, yet it has the flexibility to be mounted at depths determined by the subsurface buoy's mooring system (Figure 11) . For more details about how the SUV method works, please refer to the papers linked to under Additional Reading 32.

The SUV method may also be used for bottom-mounted deployments. The method is also better suited (than MLM) for deployments where the waves are exposed to large mean currents. Mean currents can present a Doppler shift on the wave field and introduce errors in the directional and non-directional estimates if not corrected because the transfer function used is wave number dependent (Additional Reading) 32. The SUV method does not require a correction for background currents because both the AST and the directional portion of the measurement do not have transfer functions, and so are not wave number dependent. Nortek's post-processing software packages recalculate the wavenumber when the MLMST method is used, but we recommend using the SUV method in places with known background currents to avoid any sources of error from the correction being introduced.

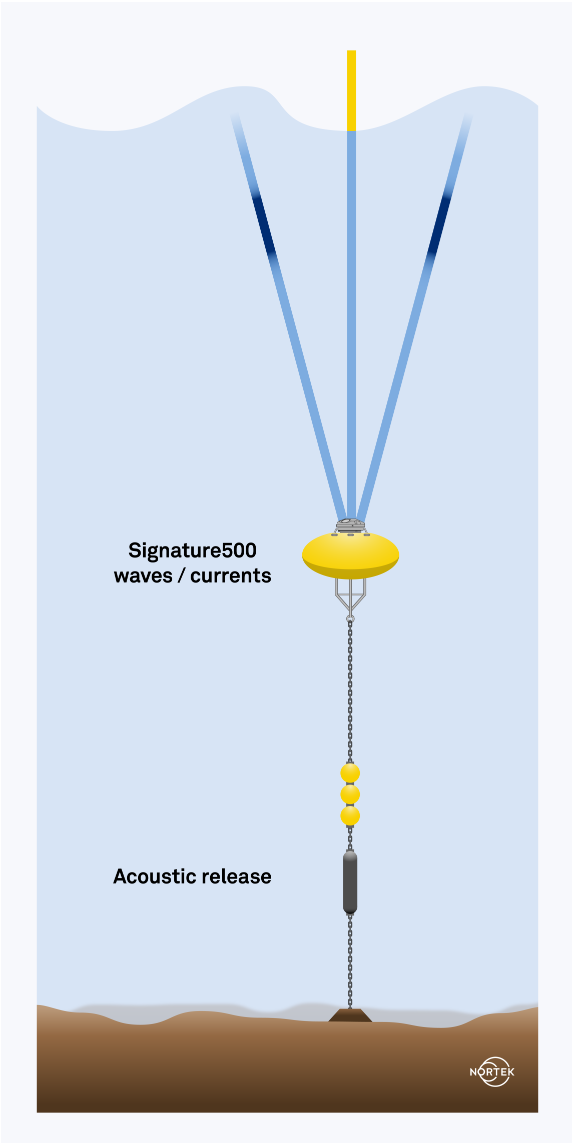

Figure 11: Directional wave measurements with AST on a subsurface buoy. Image shows location where both the slanted beams as well as the center AST Beam intersect with the water interface.

5.3.3 Limitations

Even though the introduction of AST made wave measurements possible for deeper depths than when only using the pressure, the depth still limits the minimum wavelength you are able to resolve. We distinguish between a directional and non-directional frequency limit, where the non-directional limit is lower than the directional. Both these two limitations should be kept in mind when deploying your instrument for wave measurements, and are described in detail below.

AST Footprint Limitations (Non-directional)

The ability to measure the wave parameters by use of AST is limited by the size of the area that is ensonified by the AST beam on the surface, called the AST footprint. The size of the footprint is determined by the beam width, and the instrument distance from the sea surface. The size of the footprint will increase with increased beam width or with greater distance from the surface.

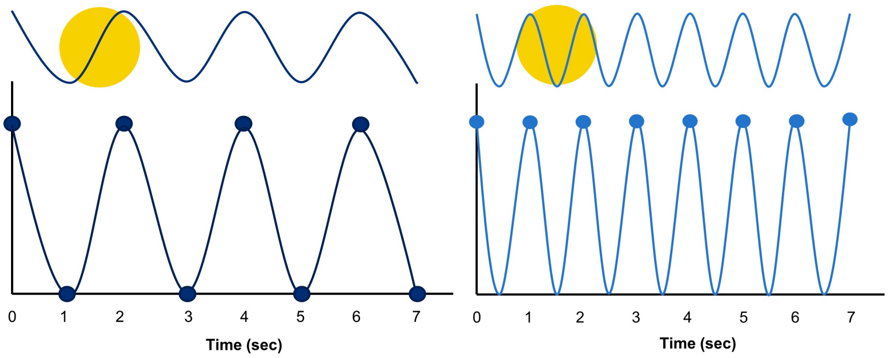

Figure 12: AST footprint limitation illustration. Blue lines indicate 2 separate water surface time series with dots representing sampling points. Yellow circles represents beam footprint rotated horizontally for ease of viewing. Blue lines indicate water surface with dots representing sampling points.

The non-directional cutoff frequency (limit of the shortest measurable wave) is affected by the size of the footprint. As a rule, we follow a Nyquist-like reasoning; the frequency limit associated with the footprint is when half the wavelength is on the order of the diameter of the footprint. This clearly is the absolute shortest measurable wave, because shorter waves will have several crests and/or troughs within the AST footprint (Figure 12). Below you will find a table that shows example deployment depth, footprint diameter, and the resulting shortest measurable wave for an AWAC. We also give an example deployment scenario for a Signature 250. The beam width for your specific instrument can be found in the technical specifications.

Depth (m)	Footprint diameter (m)	Wavelength (m)	Wave period (s)
6	0.30	0.61	0.7
12	0.61	1.22	1.0
24	1.22	2.43	1.2
36	1.82	3.65	1.5
48	2.43	4.86	1.8
60	3.04	6.08	2.0

Table 2: Minimum measurable wavelength and period based on AST footprint for an AWAC.

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The Signature250, which has a beam width of 2.3°, is to be deployed with a distance to the surface of 80 m, and the footprint on the surface from the altimeter beam will be $80 \times \sin(2.3)$. The minimum wavelength needs to be the twice of this footprint, that means ~6.42 m. One can then use simple wave theory to find an equation that relates wavelength (L) to wave period (T); $L = \frac{gT^2}{2\pi}$. Using this equation, the minimum wave period measurable at 80 m depth is ~2 s.

Directional Limitation

When using the array method to estimate the wave parameters, the directional estimates of short waves are limited by the horizontal separation distance between the cells of the projected array. This distance depend on the angle of the beams and the deployment depth of the instrument itself. As the deployment depth increases, so does the horizontal separation between individual measurement cells. Increased separation distance will lead to a larger minimum wavelength that can be resolved for directional estimates. A rough rule of thumb is that directional estimates for waves that have a wavelength that is two times the separation distance or greater can be resolved unambiguously. This aliasing presents a spatial Nyquist limit and leads to a cutoff frequency where wave directions cannot be resolved. In Figure 13 and Figure 14 both the directional and non-directional cut-off frequency or period it given as a function of deployment depth for an AWAC and a Signature 1000. Keep in mind the the wave period is the inverse of the wave frequency.

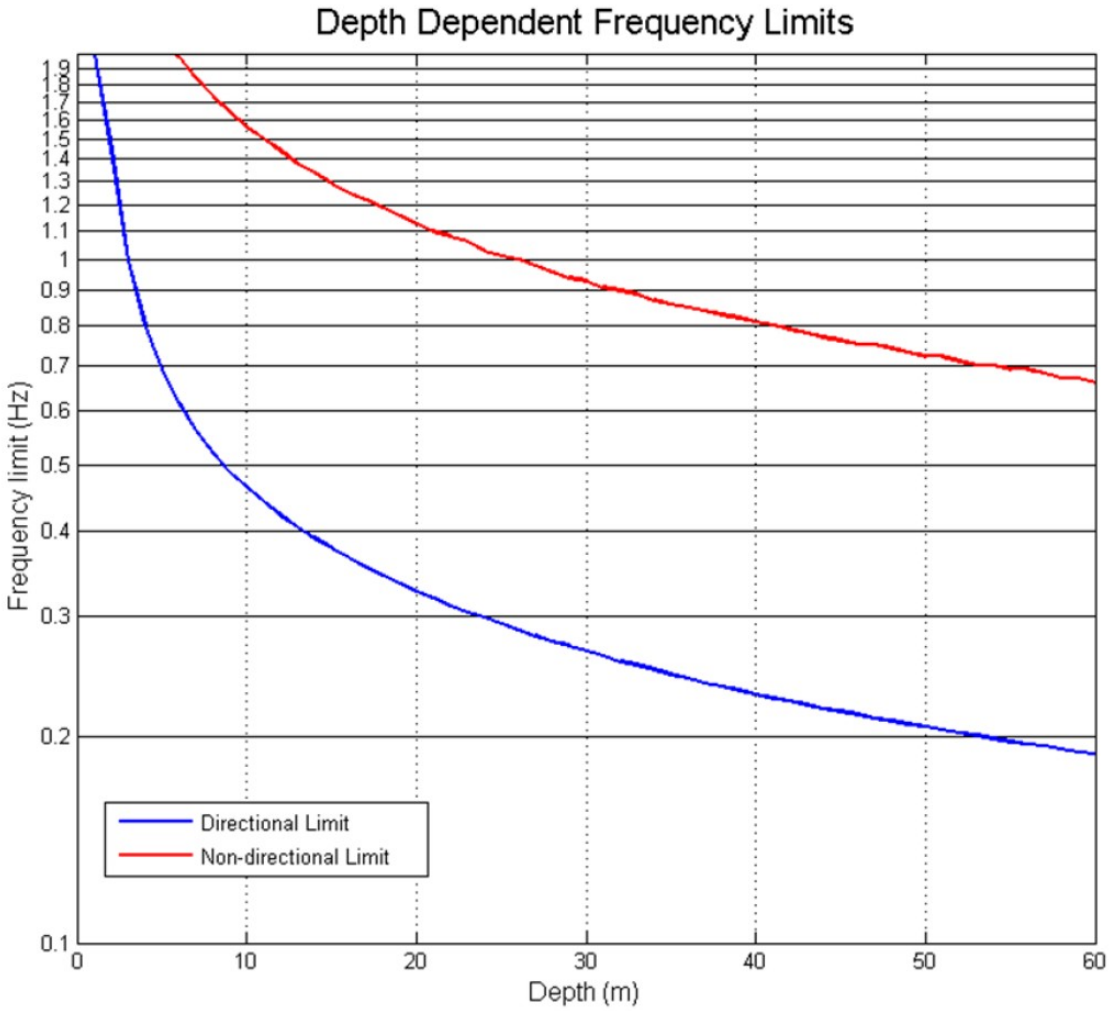

Figure 13: The cutoff frequencies for the AWAC - both for directional and non-directional estimates.

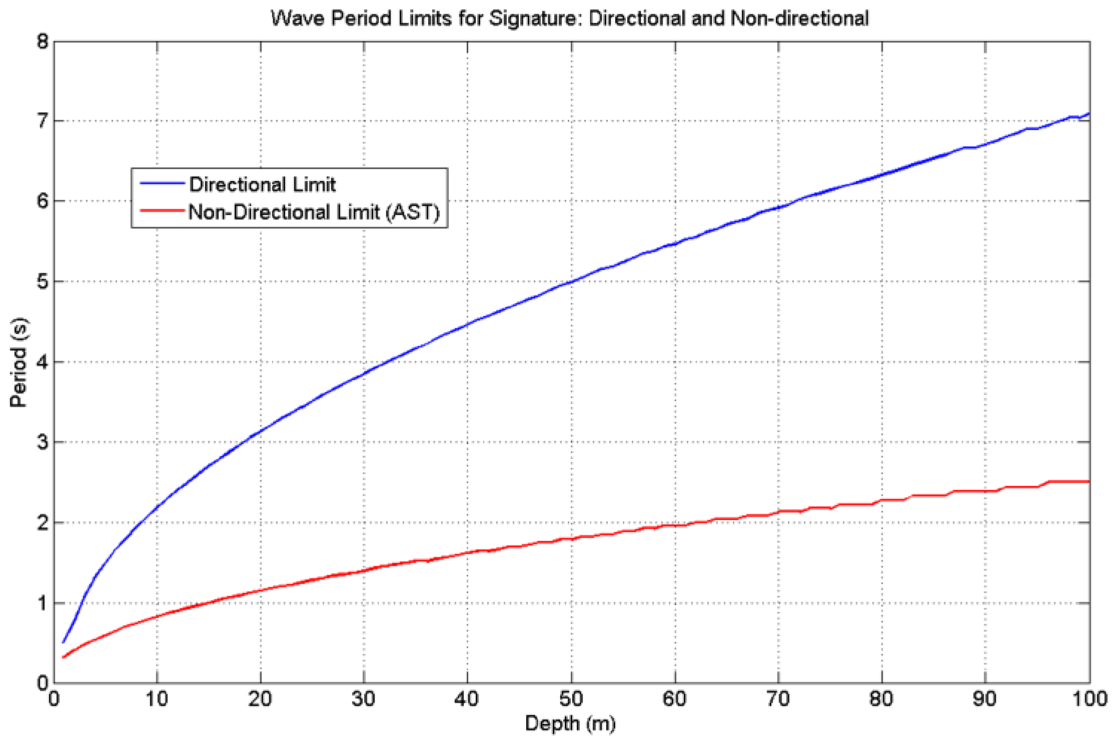

Figure 14: The cutoff frequencies for the Signature 1000 - both for directional and non-directional estimates.

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Aliasing Concept

Aliasing from an oceanographic context is the effect of overlapping wave frequencies that result in the appearance of waves that are not truly there. To visualize this concept let us consider a 10m portion of the sea surface. For this example deployment, the sampling scheme will have 8 evenly spaced surface measurements. As discussed in the Ocean waves background 5 section, it is helpful to describe the sea surface as a summation of simple sinusoidal functions. In Figure 15, the sine functions have an arbitrary amplitude of 0.25 and are offset vertically so that they can be viewed individually. The yellow points in the figure represent the points where the sea surface has been measured directly.

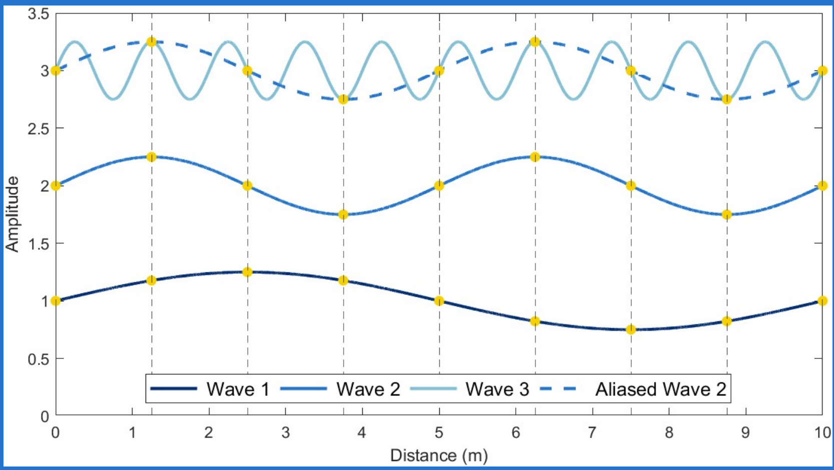

Figure 15: Representation of aliasing

It can be seen in Figure 15 that the longest possible wave in our 10m portion of sea surface is a wave with wavelength of 10m. This wave is shown as Wave 1. The smallest possible wave that can be measured over the 10m length has a period of twice the interval shown and is represented as Wave 2 in the Figure. During the deployment, the sine term with period 2deltax would be sampled at 0,pi,2pi,... At all of these points the sine function representing the wave will return to the same vertical position it began. Additionally, at all of these points the surface is still sampled, as shown by the yellow dots on Wave 3, however, the location of the sampled points for Wave 3 are at the exact same positions as Wave 2. If we were to view our deployment as a time series of surface measurements, we would not be able to distinguish between Wave 2 and Wave 3. This illustrates the phenomenon of aliasing.

5.4 Setup Considerations

The most important things to consider regarding deployment of instruments measuring waves are the focus here. What considerations that must be taken into account first and foremost depend on the wave frequency of interest, and on what processing method that can / will be used. Knowledge about the limitations that exists when measuring waves and having an anticipation of the waves that are to be captured is critical to determine the instrument's depth and ultimately successful data collection and post-processing. Therefore the aim is to achieve a "Target Depth". This is the deployment depth that will capture as much of the wave spectrum as possible with the greatest amount of accuracy. Knowing the types of waves the instrument is likely to be exposed to, and what types of waves that are of interest to measure, makes it much easier to decide upon the deployment depth.

PUV Method

Pressure, U and V velocity components (orbital velocity components) are the parameters of interest. The pressure and orbital velocity signals attenuate with depth, thus the signals from the waves are not detectable at a certain depth. If you know the significant wave height of the area and the prevailing period of the waves, you can find the depth in the table above that are able to measure the waves of interest. This table provides a general guideline for the limitations when measuring waves. Perhaps the most difficult aspect of classifying sea state via spectral based estimation is the realization of wave energy at high frequencies. These types of spectra prove to be quite tricky. Amongst the problems are the errors in the calculation of moments and cutting off peaks at higher frequency. One scenario is that an instrument placed in 15 meters of water may never detect the energy up above 0.25 Hz.

Array Method

The difference between PUV and the Array method is that the latter measure U and V closer to the surface with the consequence that shorter waves can be measured. The pressure is measured at the level of the instrument. The measurement of U and V are still limited by the frequency of the waves; as the deployment depth becomes greater, so does the horizontal separation between individual measurement cells in the surface array. In order to resolve wave direction at any given frequency, the horizontal separation of individual measurements must be less than half a wave length. This aliasing presents a spatial Nyquist limit and leads to a “cutoff frequency” where wave directions cannot be resolved. For example, a gauge deployed 40 m below the surface has a directional cutoff frequency of about 0.23 Hz, that is, a period of 4.35 seconds. This means the gauge will not be able to resolve directions from waves shorter than 4.35 seconds at 40 m depth.

MLMST and SUV Method

The same limitations of measuring U and V velocities using the Array method applies to the MLMST and SUV. The MLMST and SUV methods use the AST to measure wave energy; a method that is often more accurate because it is a direct measurement of the surface position, as opposed to the inferred pressure. The wave resolution from the AST measurements is not completely independent of deployment depth either; some high frequency wave information is lost at greater deployment depths. This limit has to do with the size of the AST footprint on the sea surface relative to the measurable wavelength. The size of this circular footprint is determined by (a) the beam width and (b) the distance from the sea surface. The size of the footprint will increase with wider beam width or with greater distance from the sea surface. The difference between MLMST and SUV is that the latter permits the instrument to rotate during the wave burst (i.e. instrument mounted on a subsurface buoy), while the MLMST method depends on measurements retrieved from a not-moving instrument.

Additional Considerations

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- Specific recommendations cannot be provided other than the sampling rate needs to be at least twice as fast as the signal of interest in order to resolve it unambiguously (i.e. the Nyquist sampling criteria).
- Mounting depth: Orbital velocities attenuate exponentially with depth and this behavior is more severe for higher frequency waves (short waves). This means that the further down in the water column that the orbital velocities are measured, the less high frequency information is available. This is the classic problem faced by bottom mounted instruments, and note that even the ADCP class of instruments suffers from this challenge if it is not managed effectively.
- Mounting angle: Since the vertical acoustic pulse is reflecting from the surface the best response occurs when the pulse path is orthogonal to the surface from which it is reflecting. The return response deteriorates as the beam deviates from the vertical. Performance is notably reduced when the tilt exceeds 5 degrees, and the data can be considered completely unusable if the tilt exceeds 10 degrees.
- Mounting method: If the instrument is to be deployed on a subsurface buoy, make sure that the natural frequency of the buoy itself is not the same of the wave frequencies being measured.

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6 Determining your Sampling Scheme

Waves are random and therefore measuring waves requires sampling over a period of time that will best “capture” or represent the complete sea state statistically. The objective is to measure the wave event over a long enough time so that the random event can be properly characterized. Sampling theory suggests, as a rough rule of thumb, that the measurement duration is long enough to capture a minimum of 100 cycles of an event (i.e. a wave). When we measure ocean waves we have to consider what the longest expected wave period is going to be; 100 cycles of this longest wave will be the minimum wave burst duration.

In order to use Nortek-developed waves post-processing software, the instrument must be set in burst mode. The number of samples should be set to 1024, 2048, or a number consistent with the chosen sampling rate to ensure over 15 minutes of data is collected. If the number of samples is set to a number not in this sequence, the software will use the next lowest sequence number, e.g. if number of samples is set to 2400, the software will only use the first 2048 samples to calculate the wave parameters.

If waves are to be measured in the Mediterranean Sea, we would expect the waves to be 10 seconds or shorter. Therefore 10 seconds x 100 cycles is 1000 seconds. The sampling rate and number of samples defines the sampling length. A corresponding configuration could be either 1024 samples at 1 Hz or 2048 samples at 2 Hz.

Sampling/Measurement Interval

The sampling or measurement interval specifies how often the instrument will collect wave data ensembles. This should be specified so that changes in the wave climate can be properly detected. Since wave events are relatively slowly developing events, choosing a relatively long interval make sense. A typical interval is every hour but may range from 0.5-3 hours depending on length of deployment time and available resources (battery and memory).

Burst Sampling

Wave data is collected in a mode that is referred to as Burst. The short version is that burst sampling mode takes rapid sampling over a specified time interval. Burst sampling is of particular use when there is an interest in sampling a specific part of the energy spectra.

Sampling Rate

This is the sample rate for the wave data measurements and it applies to the pressure and velocity measurements. The sample rate represents the absolute upper limit on the resolvable waves. When it comes to sampling rate, the general recommendation is that you will need to sample at least twice as fast as the signal of interest in order to unambiguously resolve it. The upper limit is half the sample rate; this is also known as the Nyquist Limit. You will need to make a few estimates of what time scales and processes are of interest and select a sample rate appropriate for those.

Waves sampled at 1 Hz can only resolve waves up to 0.5 Hz, and waves sampled at 2 Hz can only resolve waves as short as 1 Hz. Note that there are other mechanisms, such as deployment depth, that can have a stronger influence on the frequency limits of resolvable waves. If the sampling rate is too slow compared to the time variation of the motions, under-sampling is the result and the data is said to be "aliased" with respect to the wave motion,thus the waves cannot be well resolved.

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Number of Samples

This is the number of data samples collected during a wave burst. The optimum choice for number of samples is used in conjunction with the sampling rate where the total burst length in time is considered. As with all measurements there is a trade off between accuracy of the estimates and the consumption of resources (battery power and memory). The consumption is considerably more for wave measurements than current profiles.

7 Wave Parameters

The instruments collect raw wave data that must go through a processing step before it can be used to interpret the waves on the surface. The resulting wave parameters can be put into the two categories; non-directional parameters (wave height) and directional parameters (wave direction).

- Non-directional parameters cover parameters that do not depend on the direction of the waves, and comprise: Peak Period (Tp), Mean Period (Tm02), Mean Zero-crossing Period (Tz), Mean 1/3 Period (T3), Mean 1/10 Period (T10) and Maximum Period (Tmax), Significant Wave Height (Hm0), Mean 1/3 and 1/10 Height (H3 and H10), Maximum Height (Hmax), and Mean Height (Hmean),
- Directional parameters cover Peak Direction (DirTp), Directional Spread (SprTp), and Mean Direction (Mdir). Note that wave directions are always reported as the direction from which the waves are coming.

7.1 Non-Directional

The most common non-directional wave parameters describe height and period of the wave field and are single values representative of the time series. Period (T) is defined as the time interval between two successive peaks (or troughs) passing a fixed point, measured in seconds. The Peak Period (Tp) is the period associated with the maximum peak in the spectrum. It is found using the spectral analysis method, and it tells the characteristic frequency of the arriving wave energy (remember that frequency is the inverse of the period). The only parameter needed to find the peak period is one that varies with the wave frequency, which may be the water level, the pressure or the orbital velocity. To find the peak period, the spectral analysis is used on the time series of the parameter used to determine out the frequency with the most energy. The Mean Period (Tz) is the mean of all the wave periods in the record.

Wave height (H) is the vertical change in height between the crest and the trough. The wave height is twice the amplitude (a). The parameter most used by oceanographers to characterize a particular sea state is the significant wave height (Hs or Hm0) defined as the mean of the highest 1/3 of all waves in the record's ranking. Classically, this estimate is performed by sorting all waves in a time record according to height (referred to as Hs). However, in our approach, we utilize the spectrum of the sea surface to approximate this value. A generally accepted approximation is:

$$H_{m0} = 4.0\sqrt{m0}$$

Here *m0* represents the first momentum of the power spectrum. The k^{th} momentum is defined by:

$$m_k = \int f^k C(f) df$$

where *C* is the power spectrum, and *f* is the frequency.

Other wave height parameters of interest are the maximum wave height (Hmax), which is simply the largest measured wave in the record, and the mean of the largest 10% of all waves in the record (H10). These two parameters are commonly used for coastal design and assessment and are only possible when we have a direct measure of the surface displacement (e.g. AST). Indirect measurements of waves cannot produce these parameters; however they can be presented as linear extrapolations of Hm0 (H10 = 1.27*Hm0 and Hmax = 1.67*Hm0).

7.2 Directional

Calculated wave directions are based on the first pair of Fourier coefficients and describe the mean direction at a given frequency. The directional wave spectrum is commonly expressed as a composition of the frequency spectrum and the directional spreading:

$$E(f,\theta) = S(f) \times D(f,\theta)$$

Here, f is the frequency, θ is the direction, E is the full directional spectra, S is the energy density spectra (frequency spectrum), and D is the normalized energy spectra (directional spreading). The energy density spectrum is retrieved by spectral analysis for the time series of surface elevation (pressure, AST, orbital velocities), while the estimation of the directional spreading use a combination of the measurements taken; dynamic pressure + orbital velocities or AST + orbital velocities. The directional distribution can be approximated by a Fourier expansion according to:

$$D(f,0) = \frac{1}{\pi}\left[\frac{1}{2} + \sum_n \{a_n \cos n\theta + b_n \sin n\theta\}\right]$$

The cross-spectrum is a measure of the similarity of two different measured parameters, clarifying if they are varying together. If they vary at the same frequency, then it is likely they are related. The full cross-spectrum is presented as

$$C_{xy} = S_x S_y^*$$

where the * indicates the complex conjugate [2]. The cross-spectrum is calculated between every sensor, and the directional spectrum is assumed to be linearly related to the cross-spectrum. It has been shown that the first two pairs of Fourier Coefficients can be expressed in terms of the cross-spectrum [2]:

$$a_1(f) = \frac{C_{*u}}{[C_{**}(C_{uu} + C_{vv})]^{1/2}}$$
$$b_1(f) = \frac{C_{*v}}{[C_{**}(C_{uu} + C_{vv})]^{1/2}}$$
$$a_2(f) = \frac{C_{uu} - C_{vv}}{C_{uu} + C_{vv}}$$
$$b_2(f) = \frac{2C_{uv}}{C_{uu} + C_{vv}}$$

Here, C_{uv} represent the cross-spectra of the u and v velocity components. C_{uu} and C_{vv} are the velocity component power spectra, and C_{**} is the pressure (C_{pp}) or the surface elevation spectra (C_{ss}) (depending on the method used). Generally, the two parameters defining the directional distribution is the mean wave direction (θ_1) and the directional spreading (σ). The mean direction is expressed as:

$$\theta_1 = arctan2(b_1, a_1)$$

The directional spreading is calculated as:

$$\sigma = \sqrt{2(1 - r_1)}$$

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where

$$r_1 = \sqrt{a_1^2 + b_1^2}$$

Note that wave directions are always reported as the direction where the waves are coming from.

7.3 Measurement Errors, Uncertainties

Measurement Errors

Measurements are estimates of the value of something real. Given a real wave direction, each measurement is an estimate of this direction. If the instrument happens to be measuring waves with infinitely-long, parallel crests, wave direction is easy to define; it is perpendicular to the wave fronts. However, real waves are rarely so simple. At any given time, wave spreading blurs the wave direction, making the real wave direction meaningful only as an average.

Wave direction measurements are similar. Each direction measurement is an estimate of the mean wave direction, but when making many independent estimates, they differ from one another. Averaging many independent wave direction measurements enables you to get a better estimate of the actual mean direction of the waves you are observing. Averaging estimates together usually improves the measurement. If your estimator is unbiased, then the more you average, the better your estimate becomes. Some estimators are biased, however. No matter how much you average a biased estimator, you will always have a residual error; that is, a residual difference between your mean estimate and what you are measuring. Even so, there is value in averaging biased estimates for two reasons: 1) The bias is often smaller, or even much smaller, than the random errors you can remove with averaging. 2) If you understand the characteristics of the bias, you can correct a biased estimator to obtain a better estimate.

Uncertainty

There are three primary factors in the uncertainty:

1. The actual directional spread of the waves themselves. Uncertainty in the mean wave direction is proportional to the spreading of the waves.
2. SNR or signal/noise ratio. A noisy measurement increases the apparent spreading and the uncertainty of the measurement.
3. Averaging. Like most estimators, averaging produces more accurate estimates.

The directional estimator is unbiased, so averaging should always reduce the uncertainty. In contrast, the spreading estimator is biased. Averaging still helps, but you will always have a residual bias, the magnitude of which depends on the amount of spreading.

7.4 Wave Parameters (Processed)

Acronym	Description		Range	Note
Hm0	Calculated from energy spectrum. Known as Significant Wave Height, defined as the mean f the highest 1/3 of all waves in the record's ranking.	$Hm0 = 4.0 \times \sqrt{m0}$ *m0 represents the first moment of the power spectrum, defined as* $m_q = \int_{f_{start}}^{f_{end}} S(f) f^q df$	0-20 m	

Acronym	Description		Range	Note
H3	Time series based estimate. Mean of the 1/3 largest waves in a record.	Zero up crossing	Typically 5% smaller than Hm0	AST only
H10	Time series based estimate. Mean of the 1/10 largest waves in a record.	Zero up crossing	1.27*Hm0	AST only
Hmax	Time series based estimate. Largest wave in a record.	Zero up crossing	1.67*Hm0	AST only
Hmean	Time series based estimate. Mean values of all waves in a record.	Zero up crossing		AST only
Tm02	Calculated from energy spectrum. Mean period.	$Tm02 = \sqrt{\frac{m0}{m2}}$		
Tp	Calculated from energy spectrum. Peak period of the waves corresponding to the peak frequency.	$Tp = \frac{1}{f_{peak}}$		
Tz	Time series based estimate. Mean period. This is a direct measurement unlike the spectral equivalent, Tm02.	Zero up crossing		AST only
T3	Time series based estimate. Period associated with the 1/3 largest wave in the record H3.	Zero up crossing		AST only
T10	Time series based estimate. Period associated with the 1/10 largest wave in the record H10.	Zero up crossing		AST only
Tmax	Time series based estimate. Period associated with the largest wave (Hmax) in a record.	Zero up crossing		AST only
TpDir	Calculated from energy spectrum. Peak direction is the wave direction at the frequency at which a wave energy spectrum reaches its maximum.	Zero up crossing	0-360°	Reported as "from"
Spr1	Calculated from energy spectrum. Measure of the directional variance at peak frequency.	$spr1 = \sqrt{2 \times (1 - r_1)} \times \frac{180}{\pi}$, $r_1 = \sqrt{A1^2 + B1^2}$ *A1 and B1 are the first pair of Fourier coefficients at the peak frequency*	0-360°	

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Acronym	Description		Range	Note
Mdir	Calculated from energy spectrum. Main direction. Weighted average of all the directions in the wave spectrum.	$A1_{band} = \frac{\int S(f) \times A1(f)}{\int S(f)}$, $B1_{band} = \frac{\int S(f) \times B1(f)}{\int S(f)}$ $Mdir = atan2(A1_{band}, B1_{ban}$ *For this case the "band" is the entire frequency range of the energy spectrum*	0-360°	Reported as "from"
Unidirectivity index	Calculated from energy spectrum. Measure of how much of the wave energy over the full spectrum is from a single direction. Value of 1.0 indicates the energy is from one primary direction.		0.0-1.0	

Table 4: Processed wave parameters

8 Additional Reading

Transfer Functions

Both the dynamic pressure and the orbital velocities are driven by the surface waves. The signals associated with these properties are complicated by the fact that they attenuate exponentially with depth. The exact behavior of the attenuation has to do largely with the water depth and the wavelength. Briefly, the behavior is as follows: (1) as we move down in the water column the signal is increasingly attenuated, (2) as the wavelength decreases (shorter period or higher frequency) the signal again experiences increasing attenuation. We use linear wave theory to convert pressure and velocity spectra to surface elevation spectra. The pressure attenuation factor is given by

$$T_p = \frac{\cosh k(h+z)}{\cosh kh}$$

and for the velocity as

$$T_v = \frac{\omega \sinh k(h+z)}{\cosh kh}$$

Here, h is the water depth, z is the position in the water column, ω is the circular frequency, and k is the wavenumber. This attenuation is exactly why the instrument is restricted to measuring longer waves at deeper instrument locations. It is impossible to measure high frequency waves of low amplitudes from instruments deployed at large depths.

Cutoff and Extrapolation

A matter that must be taken into consideration when using spectral analysis to estimate wave statistics is that, as we move up in frequency there will be a point where there is no response in the signal, yet the attenuation continues to become more significant. This weakening, or attenuation, increases with frequency. At some cutoff frequency, the velocity and pressure signal becomes so weak that the waves can no longer be detected.

The problem arises when the perturbation is less than the sensitivity of the sensor. This leads to a false growth with the spectral level as we increase frequency (as illustrated in the figure below). The end result is that the calculated surface spectrum "blows up" into infinity. The reason for this false growth is that as we move up in frequency, the signal drops into the noise floor while the transfer

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function decays exponentially. Therefore, at some frequency in the spectrum a minimum must be chosen before it grows without bound.

This behavior at high frequencies necessitates the need for defining a cutoff frequency and an extrapolation from this frequency onward. Since we will ultimately integrate the spectra for the momentum calculations, we require spectra that are unambiguous and bounded. We assume that the spectrum follows a Pierson-Moskowitz or JONSWAP type spectrum. This is an empirical spectral shape, where the tail rolls off at a rate of $f^{-4.5}$.

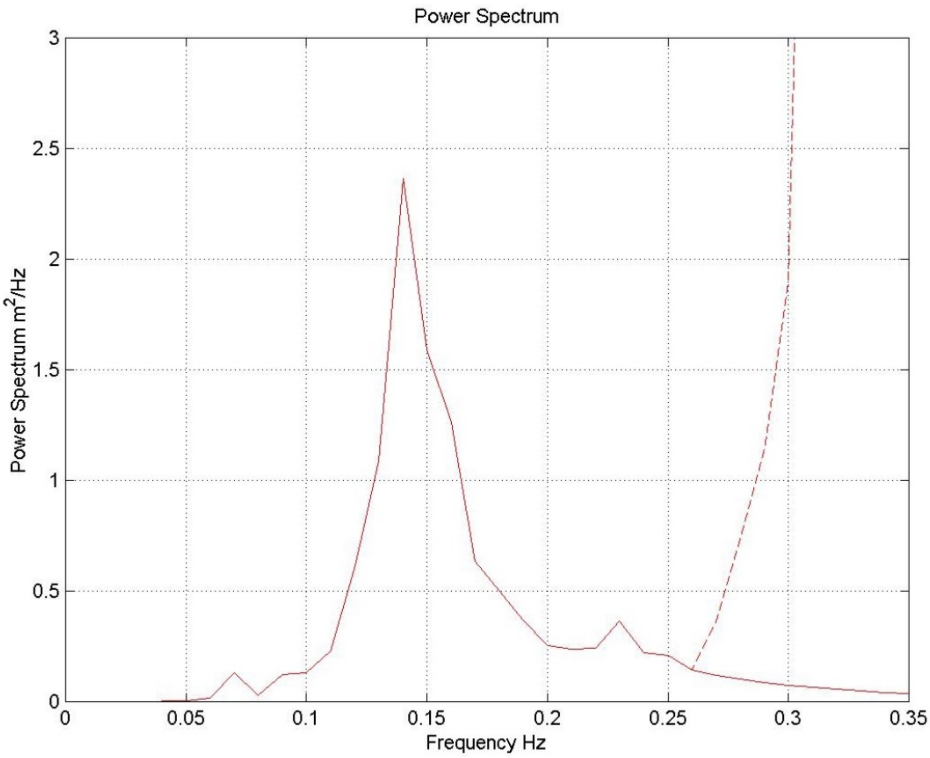

Figure 17: Example of high frequency extrapolation. Note that the original signal is represented by the dashed line. The figure is reprinted from [2].

The frequency at which the cutoff is selected is determined by finding the last local minimum above a maximum amplification factor in the spectrum as we sweep up in frequency. The problem with a sensor cutoff leaves the possibility of not detecting wave energy about the cutoff, and this lead to errors in some wave estimates (Hm0, Tm02) if no extrapolation is done. The AST has no extrapolation applied to it because it does not have a cutoff frequency limitation like the pressure or velocity based estimates. This is because AST is a direct measurement of the surface, and transfer functions are not used.

AST technology

Pedersen, T., Lohrmann, A., 2004: Possibilities and limitations of Acoustic Surface Tracking

PUV waves

Nortek AS Technical Note, Wave Measurements Using the PUV Method, (TN019), 2002: PUV Wave Measurements

SUV waves

Pedersen, T., Lohrmann, A., Krogstad, H., 2005: Wave Measurements from a Subsurface Platform

Pedersen, T. Siegel, E., 2008: Wave Measurements from a Subsurface Buoy
Pedersen, T., Horn, K., Wickström, K., 2009: Subsurface Wave Measurements Taken to New Depths

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