

Turbulent Dissipation Estimates from Pulse Coherent Doppler Instruments

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Abstract— Utilizing a commercially available acoustic Doppler velocimeter, the Nortek Vectrino with optional plus (+) firmware, measurements of turbulence are made in a turbulent open channel flow in the 8m Research Flume of the DeFrees Hydraulics Laboratory. The measurements are used to estimate dissipation (ϵ) from Kolmogorov's 2/3, 5/3 and 4/5 Laws as well as integration of the dissipation spectrum. Corrections to remove bias due to Doppler noise are carried out when appropriate. Results from the four methods are compared to judge the validity of each for use with single point velocity measurements from an acoustic Doppler Velocimeter. The 4/5 Law is the most consistent across each velocity component, but all methods produce reasonable estimates of dissipation from at least one velocity component.

Keywords—turbulence; dissipation; pulse coherent; ADV; acoustic Doppler velocimeter

I. INTRODUCTION

Because of its ubiquity in environmental flows, accurately measuring turbulence is important in many experiments. In addition to providing a more complete description of the flow, accurate turbulence measurements are essential for a variety of results. They are used to estimate mixing and dispersion coefficients for models, examine the magnitude of scalar fluxes like dissolved oxygen at the sediment water interface, and estimate sediment erosion and transport rates.

In addition to the basic statistics such as turbulent intensities and the turbulent kinetic energy, researchers are also interested in higher order quantities like the turbulent dissipation rate (ϵ). Dissipation is of interest to many researchers because it is needed to characterize the smallest scales of turbulent motion, allows closure of energy budgets, and is essential for verifying assumptions made in characterizing the turbulence.

Dissipation is difficult to measure directly because it involves gradients at the smallest scales of motion. In most flows this is at sub millimeter lengths, smaller than most instruments and techniques are capable of resolving. In order to estimate dissipation, then, researchers often turn to theoretical predictions on the form certain quantities will take, generally guided by results based on isotropic turbulence theory. A full review of isotropic turbulence theory is beyond the scope of this article. For those familiar with Kolmogorov's theory of

isotropic turbulence, the introduction of [1] provides a review of relevant calculations such as the turbulent velocity spectrum and the second and third order structure functions. A more complete review can be found in Chapters 3, 5, and 6 of [2].

The most important aspect of isotropic turbulence theory for the present discussion is referred to as Kolmogorov's Second Similarity Hypothesis. In words, it states at sufficiently high Reynolds number the turbulent velocity spectrum and the velocity structure functions take on a universal form at the intermediate scales (the inertial subrange) uniquely determined by k (the wavenumber) or r (the separation between two points) and ϵ .

Acoustic Doppler velocimeters have been shown to accurately measure the intermediate scales of turbulence by numerous researchers. The present work examines their performance in measuring the velocity spectrum and second and third order structure functions, defined in Section I.B. Relevant to the present discussion, the reader is referred to references [3, 4, 5, 6].

A. Measurements

Since its development, the acoustic Doppler velocimeter (ADV) has been utilized to make turbulence measurements. Ref. [3] performed early experiments evaluating the accuracy of the first generation of acoustic velocimeters when measuring turbulence. By comparing to laser Doppler velocimeter measurements made in the same flow, they verified the ability of the ADV to accurately measure the mean flow and fluctuating velocities.

This evaluation showed little bias in the measurement of mean velocities and co-variance terms such as the Reynolds stress. Higher bias is reported in variance terms such as the turbulence intensity. This difference in noise is a function of sensor geometry which also results in the horizontal velocity components having much higher noise than the vertical velocity. For the three-receiver system used in [3], the noise in the horizontal components are approximately thirty times higher than the vertical component.

The ADV's measurement of a turbulent velocity spectrum is also discussed in [3]. Agreement with the Kolmogorov scaling of the one-dimensional velocity spectrum is good. By applying a correction to the spectrum, better agreement between the model spectrum and the vertical velocity spectrum

is obtained. This correction estimates various sources of noise present in the measured velocities.

With the expectation the acoustic Doppler velocimeter is capable of accurately measuring turbulence, our attention turns to estimating dissipation.

B. Dissipation Calculation Methods

Utilizing hot wire measurements made in a rough wall boundary layer, [2] present a comprehensive examination of local isotropy in turbulence at an extremely high Reynolds number ($Re_\theta = 370000$, θ is the momentum thickness). Several methods are used to estimate dissipation from the hotwire data. The first method utilizes the turbulent velocity spectrum (specifically each component's one dimensional velocity spectrum) defined so that

$$\int_0^\infty E_{ii}(k_1) dk_1 = \overline{u_i^2} \quad (1)$$

Where E_{ii} is the normalized one dimensional velocity spectrum of component i , k_1 is the wavenumber in the streamwise direction, and $\overline{u_i^2}$ is the variance of the signal. There is no summation over i . Also note this integral is sometimes defined to equal $1/2 \overline{u_i^2}$, which will alter the value of the constants C_1 and C_1' defined below. One prediction of Kolmogorov's Second Similarity Hypothesis is this spectrum takes a universal form in the inertial subrange

$$\begin{aligned} E_{11}(k_1) &= C_1 \epsilon^{2/3} k_1^{-5/3} \\ E_{22,33}(k_1) &= C_1' \epsilon^{2/3} k_1^{-5/3} \end{aligned} \quad (2)$$

C_1 is a constant equal to $18/55 C$, C_1' is $4/3 C_1$, and C is Kolmogorov's constant with a value 1.5 ± 0.1 [2]. The subscript on E in the second equation implies both component 2 and component 3 meets this equivalence. This relationship is called Kolmogorov's 5/3 law.

A second form of the velocity spectrum useful for estimating dissipation is the dissipation spectrum

$$\begin{aligned} \epsilon &= 15\nu \int_0^\infty k_1^2 E_{11}(k_1) dk_1 \\ \epsilon &= \frac{15}{2}\nu \int_0^\infty k_1^2 E_{22,33}(k_1) dk_1 \end{aligned} \quad (3)$$

In this relationship, ν is the kinematic viscosity. The subscript on E implies the relationship holds for either component 2 or 3, not summation.

The second order structure function is defined as

$$D_{ii}(r) = \overline{(u(x+r, t) - u(x, t))^2} \quad (4)$$

Where r is a separation distance in the streamwise direction. The third order structure function is defined

$$D_{iii}(r) = \overline{(u(x+r, t) - u(x, t))^3} \quad (5)$$

There is no summation over i for the second or third order structure functions.

Derived from Kolmogorov's Second Similarity Hypothesis, Komogorov's 2/3 law relates D_{ii} to ϵ .

$$\begin{aligned} D_{11}(r) &= C_2 \epsilon^{2/3} r^{2/3} \\ D_{22,33}(r) &= C_2' \epsilon^{2/3} r^{2/3} \end{aligned} \quad (6)$$

Where $C_2 = 2.0 \pm 0.1$ and $C_2' = 4/3 C_2$ [2]. Like the 5/3 law, this allows measurements at intermediate scales to be used to estimate dissipation without the need to resolve the smallest scales of motion.

The third order structure function satisfies a relationship known as the 4/5 Law

$$D_{iii}(r) = -\frac{4}{5} \epsilon r \quad (7)$$

The unique property of the 4/5 law is the constant is universal and exactly determined from the derivation of this law [1]. The other constants appearing in the 5/3 and 2/3 laws are only known approximately, within about 10% of the values used here. This gives the estimates of ϵ from the 4/5 law a slight advantage from a theoretical perspective.

One final term needed when discussing dissipation is the Kolmogorov length scale, used when normalizing k and r to examine the agreement of measured spectra and structure functions with universal forms. It is defined using ϵ as

$$\eta = (\nu^3 / \epsilon)^{1/4} \quad (8)$$

For Eulerian measurements as produced by an ADV, Taylor's Frozen Turbulence Hypothesis [2] is used to transform temporal data into spatial data, with $r = Ut$ and $k = 2\pi f/U$, where U is the mean convective velocity.

C. Contribution of Doppler noise to the measured spectra and structure functions

The ADV's ability to measure turbulence is affected by the presence of Doppler noise (σ) having the following characteristics [3, 4, 5, 6]

- It has a flat spectral response at all frequencies (white noise)
- It is unbiased ($\overline{\sigma} = 0$).
- Its skew is zero ($\overline{\sigma^3} = 0$)

- It is statistically independent of the velocity fluctuations
- It is statistically independent between independent receivers

The directly measured beam velocities can be thought of as the sum of an unbiased mean velocity (\bar{b}), the turbulent fluctuating velocity ($b_i(t)$), and a component due to Doppler noise (b_{iD})

$$b_i(t) = \bar{b} + b'_i(t) + b_{iD} \quad (9)$$

Where $b_i(t)$ is the velocity measured by beam i .

Working with (9) after removing the mean component, it can be shown any term involving the velocity variance, such as the turbulence intensities and energy spectrum, is biased by the mean squared value of the Doppler noise ($\bar{\sigma}^2$) times a scaling factor related to the probe's geometry [5, 6]. It can also be shown in an ideal system, any quantity involving a co-variance is unbiased. In practical terms, no system is ideal and co-variance between components, such as the Reynolds shear stresses will be biased, but significantly less than a variance like the normal stresses.

By substituting (9) minus the mean component into (4) it can be shown the second order structure functions are biased by a term equal to $2\sigma^2$ times a geometric scaling factor.

$$D_{33}(r) = \tilde{D}_{33}(r) + 2b\bar{\sigma}^2 \quad (10)$$

The tilde represents the true value of the structure function and b is a scaling factor determined from probe geometry. In an ideal system

$$b = \frac{1}{2 \cos^2(\alpha)} \quad (11)$$

Where α is the bi-static angle. The equivalent geometric factor for the horizontal components is denoted by a and simply involves the sine squared of the bi-static angle [5].

By substituting (9) minus the mean component into (5) it can be shown the third order structure functions are unbiased by noise. Similar to the cospectrum, a co-structure function based on a velocity difference between components is noise free.

Both [5] and [6] provide means of estimating the value of $\bar{\sigma}^2$ by utilizing redundant estimates of the velocities obtained from a four receiver system. These estimates allow correction of the velocity spectrum, intensities and other turbulent quantities involving the variance. The correction outlined in [5] will be utilized to correct velocity spectra as the method in [6] is not applicable to the instrument setup used.

II. METHODS

Experiments were carried out in the 8m Research Flume of the DeFrees Hydraulics Laboratory, part of the School of Civil and Environmental Engineering at Cornell University. The flume is a recirculating type driven by two centrifugal pumps operating in parallel. The flume has a cross section of 60 cm x 60 cm and is typically operated with a still water depth of 30-50 cm. A downstream weir forces super critical flow at the outlet, while a hexagonal grid at the inlet breaks up the flow and provides decaying grid turbulence in the main channel. A small diameter brass rod is glued at the inlet to trip the boundary layer turbulent.

For the present experiments, the still water depth was 35 cm and a variety of free stream velocities were used. Discussion will focus on the highest velocity used, 40 cm/s, as it provides the highest Reynolds number.

Measurements are made using a Nortek Vectrino velocimeter with the optional plus (P) firmware. Sample rate is set to 200 Hz for all datasets, with data taken at multiple elevations. To facilitate positioning the Vectrino, it was mounted on a computer controlled stage.

The Nominal Velocity Range was set at the lowest value, in most instances 0.30 m/s, which provided good data quality indicators and a clean velocity trace in the real time software. This setup should minimize Doppler noise in the measurements. Other than altering the Nominal Velocity Range, the instrument was left in its default setup with a sample volume height of 7 mm, transmit length of 1.8 mm, and a High power level for transmit pulses. The flow was seeded with Potters Industries Spherical until adequate SNR levels were attained (SNR > 15).

Measurements were carried out for 5 minutes at each elevation, resulting in approximately 60,000 data points. The Vectrino reports three components of velocity, with redundant measurements of the vertical velocity labeled z_1 and z_2 . The redundant vertical velocity information is used to calculate noise free cospectra and co-structure functions when appropriate.

After collection and exporting to ASCII data files, the various quantities in Section I.B are calculated. No data quality screening was done on the measurements as the expectation is the averaging applied in most calculations is sufficient to minimize the influence of outliers.

III. RESULTS

At ≈ 40 cm/s free stream velocity, the momentum thickness Reynolds number is $Re_\theta \approx 6000$. This places it well below the values used by [2], but should be high enough the assumption of local isotropy is valid in this flow. At elevations away from the boundary, turbulence levels are 2-5% of the mean flow for all three components.

Measurements within the bottom boundary layer were used to estimate $u_* = (\rho/\tau_w)^{1/2}$, where τ_w is the bed stress and ρ is the fluid density, by fitting the smooth wall Law of the Wall [1] to the mean velocity profile.

For the remainder of this paper, measurements at an elevation of 8.0 cm will be utilized to demonstrate the results of each dissipation calculation method. This elevation is presumably outside the boundary layer with $z^+ = 1482$, and dominated by the decaying grid turbulence from the inlet grid.

Uncorrected, frequency based velocity spectra for all three components are shown in Fig. 1. When plotted in this manner, the inertial subrange should follow a $-5/3$ slope, plotted as a dashed line. The vertical velocity spectra follow a $-5/3$ slope at intermediate frequencies. The two horizontal component slopes do not match the predicted value due to higher noise levels.

There are two spikes in the cross-stream component, one at 30 Hz and one at 50 Hz. Because these spikes occur only in the cross-stream component and are fairly sharp, it is assumed to be a side-to-side vibration of the probe. A potential source of this vibration is from the two pumps transmitted through the flume structure, although it is unusual these spikes do not show up in the z_2 velocity which is calculated from the same beam velocities. Despite these spikes, the cross-stream velocity behavior is not severely affected.

Using the method outlined in [5], the noise spectrum N is estimated as

$$bN(k_1) = \frac{1}{2}(E_{z_1 z_1}(k_1) + E_{z_2 z_2}(k_1)) - E_{z_1 z_2}(k_1) \quad (12)$$

Where b is the geometric scaling factor determined from probe geometry defined in (10).

Corrected wavenumber spectra for the four measured velocities, along with the cospectrum $E_{z_1 z_2}(k_1)$ and the estimated noise spectrum $N(k_1)$ scaled to correct the vertical components are shown in Fig. 2. The noise correction performs as desired, eliminating noise at high wavenumbers and correcting the two horizontal spectra at mid-wavenumbers so they follow the expected $-5/3$ slope. It should be noted this

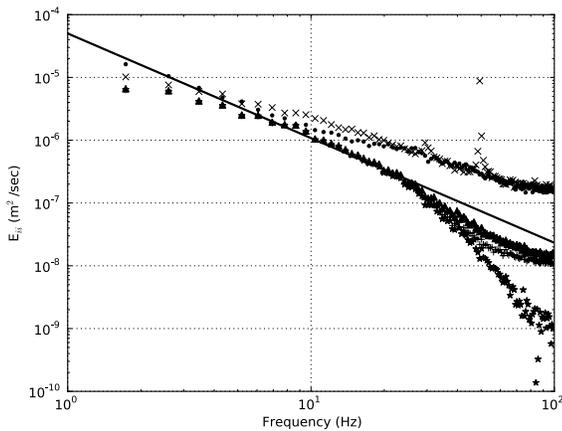


Figure 1. Uncorrected temporal spectra for each velocity component and the cospectrum for $z_1 z_2$. Streamwise (•), cross-stream (x), z_1 (+), z_2 (▲), $z_1 z_2$ (★). The solid line is a $-5/3$ slope.

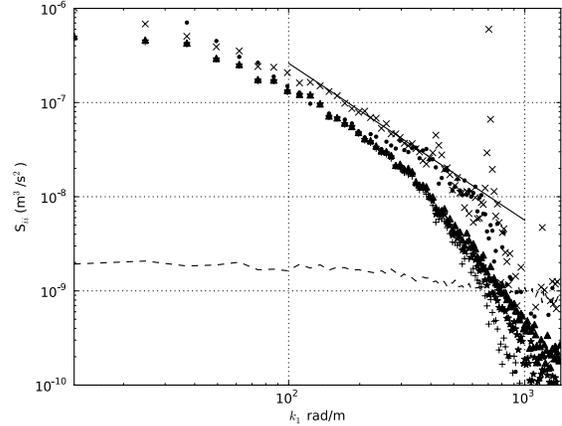


Figure 2. Corrected wavenumber spectra for each velocity component and the cospectrum of $z_1 z_2$. Streamwise (•), cross-stream (x), z_1 (+), z_2 (▲), $z_1 z_2$ (★). The solid line is a $-5/3$ slope, the dashed line is the estimated noise spectrum as defined in (12).

correction is the equivalent of the Weiner or Optimal filter and utilizes the data to develop a model for the noise. For spectral dissipation estimates, only the corrected spectra will be used.

Compensated, normalized wavenumber spectra for the four velocities and the $z_1 z_2$ cospectrum are shown in Fig. 3. Compensated spectra are defined as $E_{ii}(k) k^{5/3}$, and normalization occurs by dividing by $\epsilon^{-2/3}$. Dissipation estimates are determined by identifying the inertial subrange, which will appear as a flat region on the spectrum, on un-normalized compensated spectra. Once the inertial subrange is identified, the value of $E_{ii}(k) k^{5/3}$ is obtained for this region and used to estimate dissipation using the appropriate form of (2).

Once compensated and normalized, the spectra should take on the value of C_1 or C_1' in the inertial subrange. Because of the dissipation estimate's dependence on these constants, the

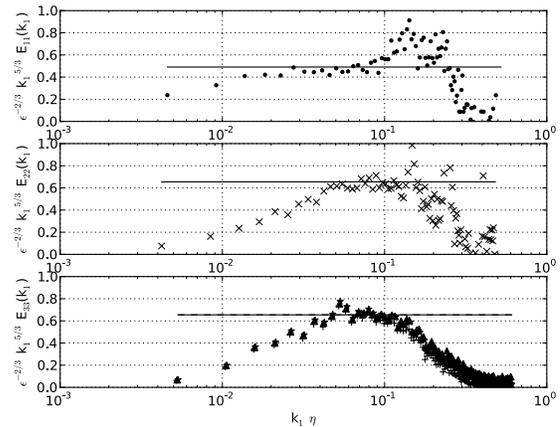


Figure 3. Compensated, normalized wavenumber spectra for each velocity component and the cospectrum of $z_1 z_2$. Streamwise (•), cross-stream (x), z_1 (+), z_2 (▲), $z_1 z_2$ (★). The solid line is has a value of $C_1 = 0.491$ or $4/3 C_1$ depending on the component being plotted.

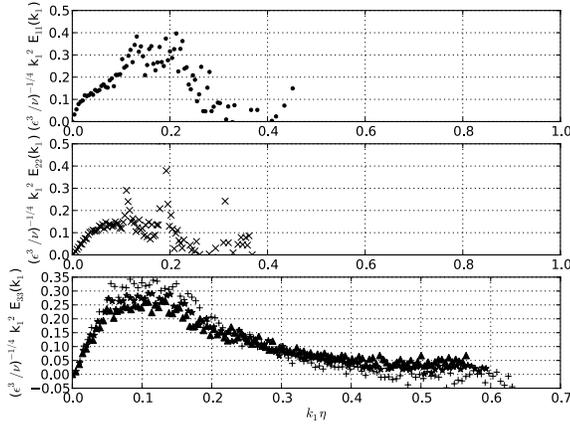


Figure 4. Compensated, normalized dissipation spectra for each velocity component and the cospectrum of $z_1 z_2$. Streamwise (\bullet), cross-stream (\times), z_1 ($+$), z_2 (\blacktriangle), $z_1 z_2$ (\star).

agreement in this normalization is excellent.

Compensated, normalized dissipation spectra are shown in Figure 4. The streamwise and cross-stream spectra are extremely noisy and not expected to yield reliable results for the integrals in (3).

The compensated, normalized third order structure functions, $(-5/4)D_{iii}/\epsilon r$, are shown in Fig. 5. Kolmogorov's 4/5 law provides a direct estimate of dissipation when the third order structure function is un-normalized by ϵ . Each velocity component's corresponding dissipation estimate is used to normalize its structure function, again making agreement with the expected form (i.e. a value of one in the inertial subrange) excellent.

The compensated, normalized, corrected second order structure functions, $(\epsilon r)^{-2/3} D_{ii}$, are shown in Fig. 5. By integrating the appropriate noise spectrum an estimate of the

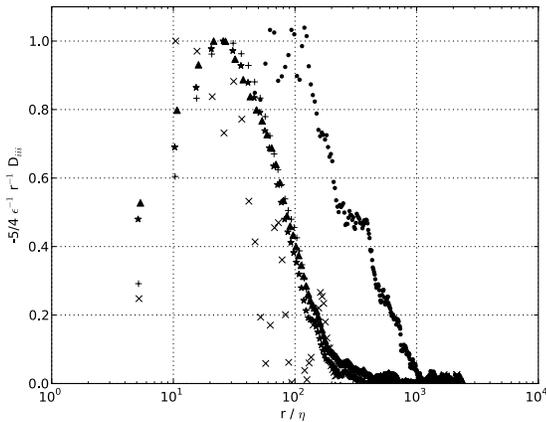


Figure 5. Compensated, normalized third order structure functions for each velocity component and $z_1 z_2$. Streamwise (\bullet), cross-stream (\times), z_1 ($+$), z_2 (\blacktriangle), $z_1 z_2$ (\star).

variance due to noise is obtained. A correction to the structure functions is applied by subtracting the noise term in (9) from the calculated structure function. When un-normalized by dissipation, the compensated value can be used to estimate dissipation from (6). Because the dissipation value estimated from this method is used in the normalization, the agreement with expected values is again excellent.

A summary of the dissipation estimates produced by the various methods for each velocity component is shown in Table I.

TABLE I. SUMMARY OF DISSIPATION ESTIMATES

	streamwise	cross-stream	z_1	z_2	$z_1 z_2$
Method	ϵ (m^2/sec^3)				
Integration	7.16×10^{-5}	2.1×10^{-4}	2.51×10^{-5}	3.92×10^{-5}	3.21×10^{-5}
5/3 Law	4.90×10^{-5}	7.28×10^{-5}	3.30×10^{-5}	3.24×10^{-5}	3.18×10^{-5}
2/3 Law	3.52×10^{-5}	3.63×10^{-5}	1.61×10^{-5}	1.75×10^{-5}	1.74×10^{-5}
4/5 Law	3.13×10^{-5}	3.19×10^{-5}	3.08×10^{-5}	3.56×10^{-5}	2.99×10^{-5}

IV. DISCUSSION

Of the methods used to estimate dissipation, the estimates from Kolmogorov's 4/5 Law are the most consistent across each component and provide the mean value closest to the estimate obtained from scaling.

The estimates from Kolmogorov's 5/3 Law are generally quite good for the three vertical spectra, but are much higher than the scaling estimate when obtained from the horizontal spectra. In particular, the cross-stream component provides an estimate over two times higher than the expected value.

Behavior of the estimates obtained from integration of the dissipation spectra is similar in behavior. Here noise unaccounted for by the correction applied to the spectra plays a larger role. The streamwise and cross-stream components yield unreliable results for dissipation. When compared to the vertical spectra, the two horizontal spectra have only the vaguest resemblance to the expected form. The vertical dissipation spectra are only resolved to $k\eta \approx 0.8$. By integrating the Pao universal spectrum [1], it can be shown this accounts for approximately 95% of the expected dissipation [7]. A correction has not been applied to these estimates.

Estimates from Kolmogorov's 2/3 Law are unusual. The estimates from the three vertical components are all approximately half of the streamwise and cross-stream values. Examining a plot of the second order structure functions reveals potential reasons for this behavior.

The inertial subrange should show as a region with a 2/3 slope. This region is apparent in the streamwise and cross-stream components, neglecting the scatter in the cross-stream data due to the vibration identified in the spectral plots. The three vertical components show only a faint hint of the 2/3 slope region in this dataset. This lack of a clearly identifiable inertial subrange renders the estimates obtained from the 2/3 Law suspect other than for an order of magnitude estimate. Further analysis examining this discrepancy is of course warranted.

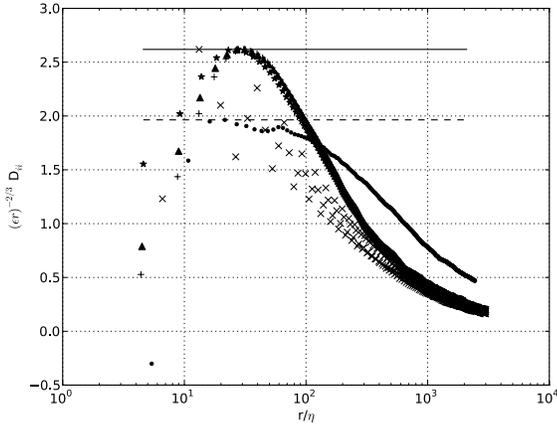


Figure 6. Compensated, normalized second order structure functions for each velocity component and z_1z_2 . Streamwise (\bullet), cross-stream (\times), z_1 ($+$), z_2 (\blacktriangle), z_1z_2 (\star). The dashed line represents C_2 while the solid line is $4/3 C_2$.

The spectral and second order structure function estimates have one main disadvantage when utilized with data from an ADV. They suffer bias due to noise inherent in ADV measurements. If uncorrected, this bias increases the uncertainty in dissipation estimates beyond the 10% level associated with uncertainty in the constants. Noise contamination is a significant issue for horizontal spectra. Unless an order of magnitude estimate is needed, correction of the horizontal spectra must be carried out.

As previously mentioned, the 4/5 Law provides the most reasonable and consistent results of any method utilized here. Given the constant involved is universal and exactly known and the unbiased estimate of it obtained from ADV data, it is the preferred method for estimating dissipation when an

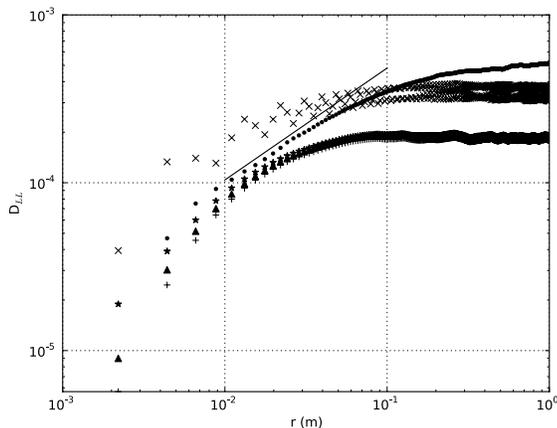


Figure 7. Second order structure functions for each velocity component and z_1z_2 . Streamwise (\bullet), cross-stream (\times), z_1 ($+$), z_2 (\blacktriangle), z_1z_2 (\star). The solid line is a 2/3 slope.

assumption of isotropy is reasonable. It should be noted behavior on the third order structure functions for the non-streamwise components is not routinely published. Despite the excellent agreement seen here, when making estimates using the 4/5 Law, the streamwise component should be the primary component examined.

In terms of calculation, there are tradeoffs with each method. The spectral calculations are simple and quick to carry out when using the Fast Fourier Transform. Care must be taken to ensure spectra meet the normalizations specified in (1) and adjust the constants in (2) appropriately depending on what the normalization integral in (1) evaluates to. The structure functions are slower to calculate, but have an advantage over the spectral methods in that confidence intervals are trivial to calculate via the bootstrap, if potentially time consuming.

V. CONCLUSIONS

Four general methods were used to estimate dissipation from a velocity time series. They include direct integration of the dissipation spectrum, use of Kolmogorov's 5/3 law to scale the one dimensional velocity spectra, and use of Kolmogorov's 2/3 and 4/5 laws to scale the second and third order structure functions.

Because estimates of the third order structure function calculated from ADV velocity measurements are unbiased, it is the preferred method for estimating dissipation.

The results of this experiment show the acoustic Doppler velocimeter can be used for estimating dissipation provided general measurement quality is satisfactory and the assumption of local isotropy is valid for the flow.

REFERENCES

- [1] P.A. Davidson, *Turbulence: An Introduction for Scientists and Engineers*. New York, New York: Oxford University press, 2004.
- [2] S.G. Saddoughi and S.V. Veeravalli, "Local isotropy in turbulent boundary layers at high Reynolds number," in *Journal of Fluid Mechanics*, vol. 268, pp. 333-372.
- [3] G. Voulgaris and J.H. Trowbridge, "Evaluation of the acoustic Doppler velocimeter (ADV) for turbulence measurements," in *Journal of Atmospheric and Oceanic Technology*, vol. 15, pp. 272-289, 1998.
- [4] A. Lohrmann, R. Cabrera, and G. Gelfenbaum, and J. Haines, "Direct measurements of Reynolds stress with an acoustic Doppler velocimeter" in *Proc. IEEE Fifth Conf. on Current Measurements*, St. Petersburg, FL, IEEE Oceanic Engineering Society, pp 205-210, 1995.
- [5] D. Hurther and U. Lemmin, "A Correction Method for Turbulence Measurements with a 3D Acoustic Doppler Velocity Profiler," in *Journal of Atmospheric and Oceanic Technology*, vol. 18, pp. 446-458, 2001.
- [6] K. Blanckaert and U. Lemmin, "Means of noise reduction in acoustic turbulence measurements," in *Journal of Hydraulic Research*, vol. 44, pp. 3-17, 2006.
- [7] E.A. Cowen and S.G. Monismith, "A hybrid digital particle tracking velocimetry technique," in *Experiments in Fluids*, vol. 22, pp. 199-211, 1997.