Direct Measurements of Reynolds Stress with an Acoustic Doppler Velocimeter

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[1]**Abstract - Ocean boundary layer dynamics is dominated by high-frequency, intermittent phenomena that can only be described in detail using turbulent statistics. Unfortunately, direct measurements of bulk turbulent parameters (Kinetic energy, dissipation rates, Reynolds stress, etc.) have proven elusive because precise estimates require current meters that can measure all three velocity components, have a good 3D response, are reasonably accurate, and have a sampling rate that is high enough to capture the scales of interest.**

As a candidate sensor for ocean deployment, the SonTek ADV was tested in a flume located at the University of South Florida. In the test, data were collected simultaneously with a 2D LDV and the 3D ADV in a collocated sampling volume. Comparison of the Reynolds stress over a velocity range of 5-80 cm/s shows a very good correspondence at high flows and a slight bias in the ADV data at low flows (<10 cm/s).

An array of seven ADVs was deployed as part of Duck '94, a large-scale field project taking place in the fall of 1994 off Duck, North Carolina. The sensors were mounted on a vertical mounting frame and deployed at a depth of approximately 4 m. In this paper, we discuss methods for extracting the turbulent properties of the joint wave-current field using an ADV. The data examples are taken from a storm event at Duck with 2.2 m significant wave height on November 7, 1994.

I. INTRODUCTION

Boundary layer dynamics tend to be dominated by rapid momentum fluxes through intermittent, convective processes. The most common parameterizations of these momentum fluxes are based on the eddy-diffusivity concept in which the turbulent shear stresses in the Navier-Stokes equations are related to the mean velocity shear by use of an eddy viscosity coefficient. This mixing constant may be a function of depth only, a function of depth and bottom stress, or a function of the detailed characteristics of the flow field such as turbulent kinetic energy or turbulent dissipation. Direct measurements of all these parameters, however, have proven elusive because precise measurements of the Reynolds stresses (u'w', v'w', etc.) require current meters that can measure all three velocity components, have a good 3D response, are reasonably accurate, and have a sampling rate that is high enough to capture most of the energy.

The Acoustic Doppler Velocimeter (ADV) is a remote-sensing, three-dimensional (3D) velocity sensor, originally developed and tested for use in physical model facilities. Implemented as a "bistatic" (focal-point) acoustic Doppler system, the ADV has the advantage of being inherently drift-free and does not require routine recalibration. Also, acoustic pulses do not suffer the range limitations of optical pulses in turbid water.

In the present implementation [1,2], three 10-MHz receive elements are positioned in 120° increments on a circle around a 10-MHz transmitter. The probe is submerged in the flow and the receivers are slanted at 30° from the axis of the transmit transducer and focus on a common sampling volume. The volume is located either 5 or 10 cm from the probe to reduce flow interference. The 3D velocity is measured at a rate of 25 Hz in a sampling volume of less than 1 cm^3. The relatively high temporal resolution and small sampling volume makes the ADV a candidate sensor for measuring field and prototype scale turbulence on a routine basis.

Fig. 1 shows a 2D simplification of the basic measurement technique. The system operates by transmitting short acoustic pulses along the transmit beam. As the pulses propagate through the water, a fraction of the acoustic energy is scattered back by small particles suspended in the water (e.g., suspended, sediments, small organisms, etc.).

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[1] The development of the Acoustic-Doppler Velocimeter (ADV) was initiated under contract by the U.S. Army Engineer Waterways Experiment Station (WES).

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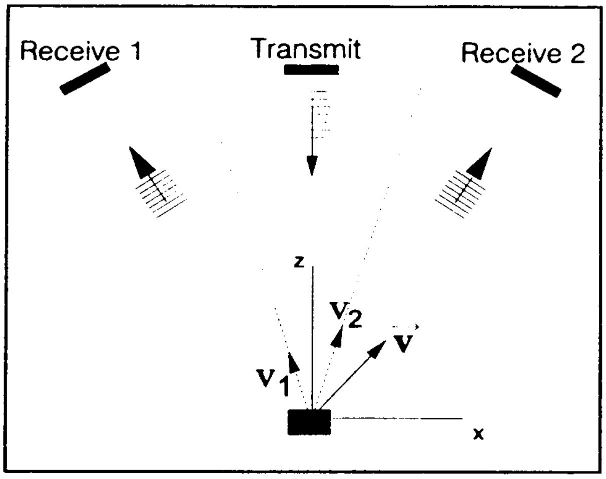

Fig. 1 - Bistatic Doppler system

The three receivers detect the "echoes" originating at the sampling volume, which are Doppler shifted due to the relative velocity of the flow with respect to the probe. The Doppler shift observed at each receiver is proportional to the component of the flow velocity (V_1, V_2, and V_3) along the bisector of the receive and transmit beams. To derive the orthogonal components of the water velocity vector V, we express the geometry of the probe as a 3 by 3 transformation matrix **A**:

$$V = A\ V_i \tag{1}$$

where V_i are the three measured velocities.

II. ESTIMATION OF TURBULENT PARAMETERS

Of special interest is the possibility of extracting turbulent quantities such as Reynolds stress, turbulent kinetic energy, and dissipation rates. For the purpose of analyzing our ability to accurately estimate these parameters with an ADV, we separate the measured velocity components V_i into a "mean" velocity and a fluctuating part that contains turbulent energy V_i' and Doppler noise V_{di}:

$$V_i = \overline{V}_i + V_i' + V_{di}\ , \qquad \overline{V_i' + V_{di}} \equiv 0 \tag{2}$$

The mean and fluctuating part of the velocity are attributable to the fluid motion whereas the Doppler noise is associated with the measurement process itself. The term is an inherent part of all Doppler-based volume backscatter systems. In a properly implemented Doppler system it has the following characteristics:

a) It has zero-bias, i.e., the estimate of the mean can be improved by averaging over independent realizations.
b) It is uncorrelated with the velocity fluctuations
c) The noise spectrum is flat ("white noise").
b) The noise fluctuations in two independent channels are uncorrelated.

The possibility of reducing the noise by averaging is important because it means that the importance of Doppler noise will diminish as the averaging period increases. The independence of the noise term with the velocity fluctuations (c) leads to the following relationship:

$$\overline{(V_i' + V_{di})^2} = \overline{V_i'^2} + \overline{V_{di}^2} \tag{3}$$

This is an important aspect of the noise and allows the calculation of turbulent quantities that are smaller than the Doppler noise.

Combining (1) and (2), the turbulent kinetic energy can be expressed as:

$$|V'^2| = \sum \overline{(AV_i)'^2} + \sum \overline{(AV_{di})^2} \tag{4}$$

The Doppler noise enters directly into the equation and the measured variance will, in general, be biased high. Because the Doppler noise is white it is easily identified as a "noise floor" in the spectrum (see Fig. 2) and its signature is a flattening of the energy slope as we approach the Nyquist frequency (f_N) at 12.5 Hz. In the worst case, this may take place round 1-4 Hz for the horizontal component, but more typically in the range 5-10 Hz. For the vertical velocity, the

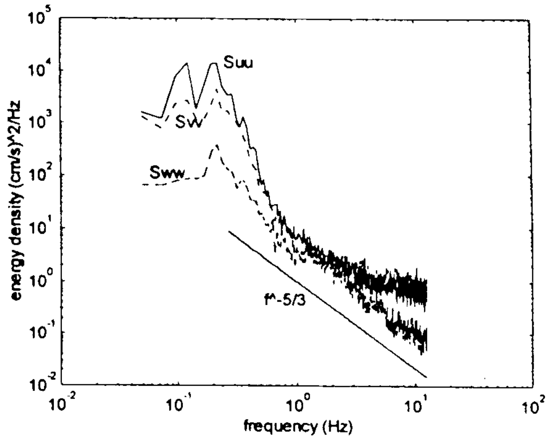

Fig. 2 - Typical power spectrum for the ADV. The Doppler noise level is higher for the horizontal components Suu and Svv than for the vertical component Sww. The dissipation range follows the -(5/3) slope. Data from Duck '94 (November 7).

noise level rarely exceeds the turbulent fluctuations when sampling at 25 Hz. The large difference between the horizontal and vertical components is a result of the probe geometry as expressed by the transformation matrix **A**. In some cases it is possible to take advantage of this difference and estimate the dissipation levels indirectly by identifying the start of the spectral equilibrium range for the vertical velocity component. [4].

The Reynolds stresses are typically smaller than the turbulent kinetic energy level and sometimes smaller than the Doppler noise. If the receive beams were orthogonal, i.e. V_i=**V**, this would not be a problem because the Doppler noise in two independent channels is uncorrelated and the estimate of the Reynolds stress would be independent of the magnitude of the Doppler noise. In the case of an ADV, however, the receive beams are not orthogonal and Reynolds stress must be derived using the matrix **A**. If we let the x, y, and z-coordinates be defined by the probe geometry, we can write:

$$\begin{aligned}\overline{u'w'} = {} & a_{11}a_{31}(\overline{V_1'^2}+\overline{V_{d1}^2})+a_{12}a_{32}(\overline{V_2'^2}+\overline{V_{d2}^2})+a_{13}a_{33}(\overline{V_3'^2}+\overline{V_{d3}^2})+ \\ & (a_{11}a_{32}+a_{31}a_{12})\overline{V_1'V_2'}+(a_{11}a_{33}+a_{31}a_{13})\overline{V_1'V_3'}+(a_{12}a_{33}+a_{32}a_{13})\overline{V_2'V_3'}\end{aligned} \tag{5}$$

The first line contains three terms that all involve the total variance of the measured velocity. The second line only contains terms with cross-products between channels and the uncorrelated Doppler noise has no contribution (Eq. 4). If we insert the actual values from the matrix A we find that the sum of the constants in the first line is zero ($a_{11}a_{31}$ + $a_{12}a_{32}$ + $a_{13}a_{33}$ = 0). This implies that the stress estimate is unbiased if the Doppler noise is equal in all three channels.

III. LABORATORY TESTING IN FLUME

In an experiment conducted in a flume located at the University of South Florida, a Dantec Laser Doppler Velocimeter (LDV) and a SonTek ADV were positioned such that the sampling volume of the LDV was located inside the sampling volume of the ADV. The laser beams penetrated the plexiglass walls and were focused at point located somewhere between 5 and 15 cm above the bottom, varying from run to run. The ADV was mounted from above, with the sensor head located 5 cm above the intersection point for the laser beams. Data were collected at 25 Hz with the ADV and 50 Hz with the LDV over a velocity range from 5 to 80 cm/s. A series of statistical comparison tests were conducted with good overall agreement for the mean horizontal flow and the horizontal variance.

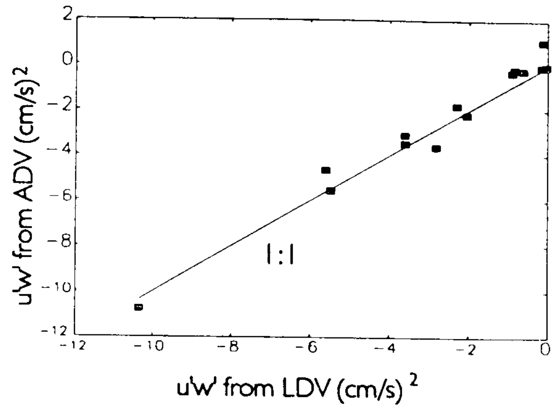

Fig. 3 - Turbulent stress measurements from flume with mean velocities varying from 5 to 80 cm/s. The straight line depicts a 1:1 relationship between the LDV and the ADV.

For the comparison of the Reynolds stress, the overall agreement was quite good (Fig. 3). The contribution to the stress was evenly divided between the first and the second term in (5), implying that asymmetries in the along-beam velocity fluctuations are important. At low flows (<10 cm/s), the ADV had a slight positive bias This offset was found to be caused by variation in the sensitivity of the three ADV receivers that lead to differences in the magnitude of the noise terms. At higher flows, the importance of the noise term becomes negligible and the match between the data collected with a Dantec LDV and the ADV is quite good.

IV. DEPLOYMENT AT DUCK, NORTH CAROLINA

A vertical array of seven ADVs was deployed in a nearshore experiment taking place off Duck, North Carolina. The array of 3D velocity sensors was located at a depth of approximately 4 m, with the lower sensors spaced 20-25 cm apart and the upper sensors spaced approximately 75 cm apart. Two pressure sensors, an acoustic altimeter, and two light-scattering sensors were also mounted to the deployment tower (Fig. 4). The ADV processor modules were located in an underwater computer and the data transferred real-time to shore using ethernet communication cards.

All the instruments were mounted to a single pole that was connected through a U-joint to a steel pole that was jetted 5 m into the bed. The upper pole was secured in a vertical position using three guy lines attached to legs that extended 2.5 m from the base of the pole below the U-joint.

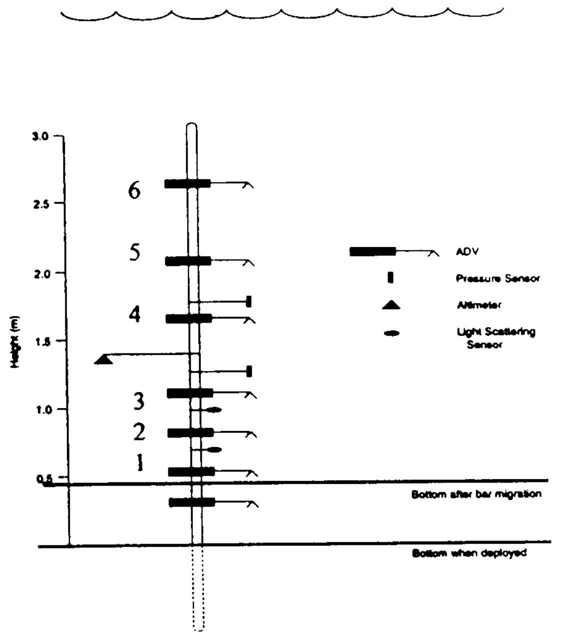

Fig. 4 - The deployment frame with seven ADVs. On November 7, the lowest instrument was buried in the sand and the sensor located 0.5 m above the original bottom (sensor 1) was too close to the bottom to yield reliable results. Sensors 2-5 were working well without any observed drop-outs. Sensor 6 (top) had some ambiguity errors [3] during the peak of the storm.

The ADV sensor head was mounted on a horizontal stem that was reinforced using 1/2" tempered steel rods that were fixed onto an extra protective ring to withstand the anticipated storms events (Fig. 5). The sensor head itself was oriented downward such that the sampling volume was positioned below the assembly. The purpose of this orientation was to minimize the possibility of flow interference from the sensor into the sampling volume.

Initial problems with the power supply in the underwater computer resulted in a delay in the deployment of the complete system. During the delay period, a large storm from the northeast changed the bottom topography and the two lower ADVs were covered by a new sand bar. The deployment construction, however, withstood the storm quite well and all the sensors survived the 3 m high waves. For a 4-week period after the storm, data were collected continuously at a rate of approximately 300 Mb per day with all ADVs sampling continuously at 25 Hz. Data collection was ended when hurricane Gordon passed near the area and floating debris broke the contact between the sensors and the underwater computer.

Several smaller storms passed through the area during the data collection period. Example data from the November 7 are shown in Fig. 6-8. The significant wave height offshore was 2.2 m and the wind speed 15 m/s.

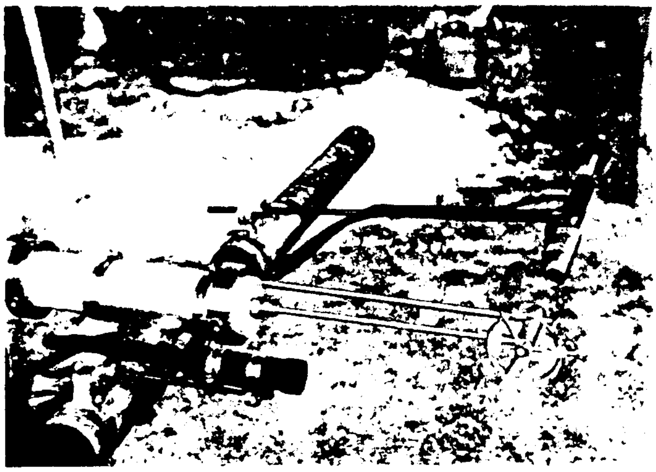

Fig. 5 - Photograph of ADV mounted horizontally on deployment frame. The sensor has a 10-MHz acoustic transmitter in the middle and three receivers positioned on a ring around the transmitter. The sampling volume is located 10 cm below the sensor.

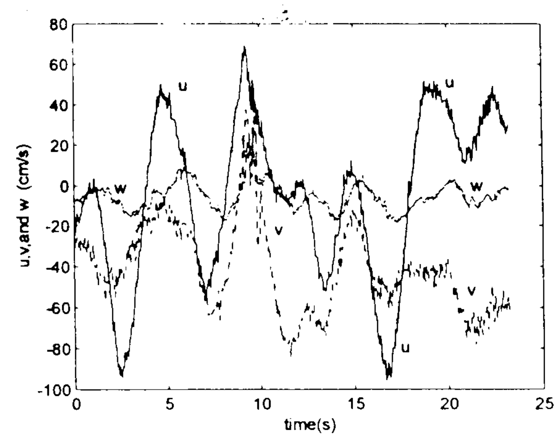

Fig. 6 - Time series of u (solid) ,v(dotted), and w (dashed) from ADV at level 3, November 7, 1994.

V. CORRECTION FOR TILT

In a wave-environment, the correction for tilt is critical for direct calculation of Reynolds stress. In the absence of tilt correction, the vertical and horizontal velocity variance will contaminate the estimated stress in proportion to the tilt angle α (first order). If we denote the average, measured values with a bar and let the expected value (<>) signify the correct values in the coordinate system where we seek to describe the physical processes, we can write:

$$\overline{u'w'}=\langle u'w'\rangle + \alpha(\langle w'w'\rangle-\langle u'u'\rangle) \qquad (6)$$

Near the bottom, the wave orbits are strongly elliptical and typical values during a storm at Duck were 25 $(cm/s)^2$ for <w'w'> and 2000 $(cm/s)^2$ for <u'u'>. This implies that the tilt angle must be determined by a precision of 0.03° if the estimated value <u'w'> is to be correct within 1 $(cm/s)^2$.

The naive approach to tilt correction is to simply rotate the coordinate system around the vertical axis until the mean flow is zero along one horizontal axis and maximum along the other. The tilt angle is then found explicitly by rotating the velocities into a coordinate system where <w> is zero. The problem with this method is that it only estimates the tilt in the direction of the mean flow. Another approach is to rotate the coordinate system around the vertical axis until the horizontal variance is maximum along one axis and minimum along the other. The tilt angle is then found by rotating the coordinate system around the axis of minimum energy and apply a curve-fitting algorithm to find the angle at which <w'w'> has a minimum. Unfortunately, this procedure is mathematically equivalent to setting <u'w'> to zero.

The method we are currently refining describes tilt as a result of a rotation β around the x-axis and a rotation α around the y-axis. The complete transformation between the measurement system and the new coordinate system can then be described as the product of the two rotation matrices [5]:

$$\begin{aligned} \langle w\rangle &= f\,(\alpha,\beta,\overline{(u)},\overline{(v)},\overline{(w)})=0 \\ \langle w'w'\rangle &= g\,(\alpha,\beta,\overline{(u^2)},\overline{(v^2)},\ldots\ldots,\overline{(v'w')})=\mathit{Minimum} \end{aligned} \qquad (7)$$

The first criterion states that the mean vertical velocity in the new coordinate systems is zero. The second criterion requires that the variance of the vertical velocity has a minimum. This is based on assumptions about the wave propagation and can be modified to include energy contributions only from the surface waves.

As a practical matter, we can substitute α=h(β) in the equation for <w'w'> by solving the function f explicitly for α. The best fit β_0 can then be found by evaluating the function g for a series of values and applying a parabolic curve-fitting equation. After performing this calculation on several data sets from November 7, 1994, the tilt angles were found to be:

Level:	2	3	4	5	6
α	5.50	2.90	1.57	1.81	2.17
β	5.58	8.44	4.16	0.07	2.29

Variations in the results based on different data segments from the same day were typically 0.1°.

VI. PRELIMINARY RESULTS

As of time of writing, the altimeter data had not been processed and the lack of a reference for elevation ruled out the possibility of comparing shear stress calculated from velocity profiles with direct estimates of Reynolds stress. Even if substantial data analysis work remains, however, some basic question about the performance of the ADV can be answered. First, the ADVs show very consistent results from one level to another. If we, for example, calculate S_{uu} for all levels, the spectra are almost indistinguishable from one level to another on the scale used in Fig. 2. Second, the measurements of <u'w'> are consistent across all levels and seem reasonable - although a closer look at the physical processes is required in order to make a firm judgement.

In Fig. 7, we have plotted the instantaneous value of -u'w' from the ADV at level 3. As can be seen, the amplitude of the signal varies quite rapidly. The wave frequency is clearly dominant with short time-scale events superimposed.

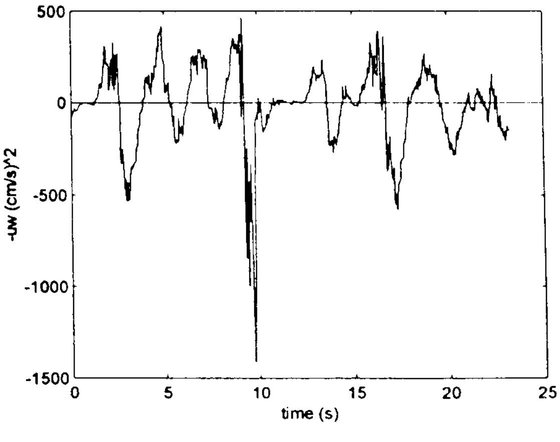

Fig. 7 - Instantaneous values of -<u'w'> after correction for tilt. The data are based on the time series in Fig. 6

In Fig. 8. we have plotted the accumulated frequency contribution to -<u'w'>. This variable is defined as the integral over the cross spectrum S_{uw}:

$$-\langle u'w'\rangle_{f_0} = \int_0^{f_0} S_{uw}\,df \tag{8}$$

The results from this particular storm event show quite consistently that the contributions from the wave-band around 0.07-0.1 Hz and from the wave-band around 0.15-0.25 Hz have opposite sign. The two contributions almost cancel each other and the net contribution from the wave-field to the stress is quite small at all vertical levels.

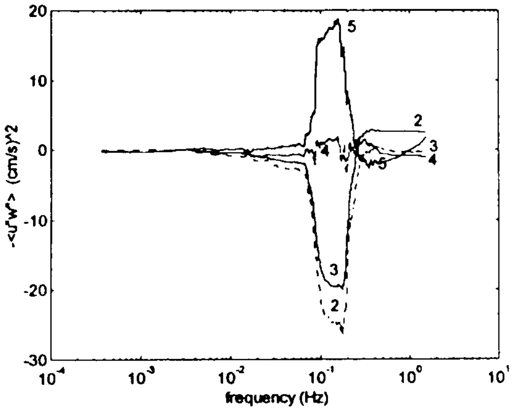

Fig. 8 - Accumulated contribution to -<u'w'> from Suw

In the high-frequency region (f>0.4 Hz), the measured contribution to the stress is quite small and the integrated contribution from the band 1-12.5 Hz (not shown in Fig. 8) is less than 1.5 $(\mathrm{cm/s})^2$ at all levels. We interpret this to mean that the fluctuations - even if they take place at very short time scales in the time series - are non-stationary in the statistical sense and do not automatically appear as high frequency phenomena in the spectral analysis.

VII. CONCLUSIONS

For moderately energetic environments, the SonTek ADV has the resolution and response required to allow routine measurements of turbulence. For turbulent kinetic energy, the sensitivity is limited by the Doppler noise and for spectra of arbitrary shape a simple integration will tend to overstate the energy level. For spectra where the dissipation range can be resolved within the Nyquist frequency (12.5 Hz), other methods based on identifying the beginning of the saturation range can be used. These methods can also take advantage of the significantly lower noise level in the vertical component than in the horizontal components.

For the Reynolds stress, the sensitivity is not limited by the Doppler noise but by how well the receive sensitivity is balanced in the three channels. Comparison with LDV in flume shows good agreement for mean flows above 10 cm/s and this limit can be brought down to 3-5 cm/s in a properly balanced system. In high-energy environments - and especially in oscillatory flow - our ability to correctly measure Reynolds stresses is limited by the requirement to correct for instrument tilt. The precision required to make the correction is quite high and data from an external tilt sensor cannot be expected to give satisfactory results. Instead, methods that depend on minimizing properties such as the mean vertical velocity and the vertical variance in the wave band must be further refined.

Preliminary results from seven ADVs deployed at 4 m depth Duck in September-November 1994 show the data to be consistent at all levels and the spectral characteristics appear reasonable. Further work will be required to numerically evaluate the results of the tilt corrections and to interpret the results in the context of the physical processes taking place in the nearshore environment.

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