Validation of a New Generation DVL for Underwater Vehicle Navigation

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***Abstract*—Nortek has developed a new Doppler velocity log (DVL) based on a novel bottom detection principle. This allows for the ability to estimate individual beam and ping Doppler measurement noise in real time. Another capability that has been embedded in the new DVL is an accurate timing reference of the velocity estimate. Using the Kongsberg Maritime (KM) HUGIN autonomous underwater vehicle (AUV) as testbed, we present in this work sea trial results for the new Nortek DVL.**

I. INTRODUCTION

Prerequisites for a state-of-the-art underwater inertial navigation system (INS) include: (1) integration of a navigation grade inertial measurement unit (IMU), having excellent gyrocompass capability when paired with the INS; (2) a versatile and robust suite of external aiding sensors; (3) accurate time stamping of the data from the IMU and the different aiding sensors; (4) accurate sensor installation and calibration; and (5) suitable navigation post-processing tools in order to further enhance precision and accuracy.

While a wide range of different aiding sensors and techniques exist [1], [2], [3], [4], the backbone in an INS applied for autonomous underwater vehicle (AUV) navigation has been, and remains, the bottom-track velocity data from a Doppler velocity log (DVL). This is particularly the case during autonomous missions [5], [6] where external positioning is sparse, and the INS is left with external velocity information as the main aiding source. If within sensor range, the DVL measures the vehicle linear velocity relative to the seabed. While not discussed any further in this paper, alternative velocity aiding techniques are described in [7], [8], [9].

As for the impact the DVL has on the overall navigation performance, this depends both on the specifications and features of the DVL, as well as the fidelity of the system integration [10]. The amount of literature on error sources in DVL based navigation is extensive. For DVL aided INS the horizontal position error drift is determined by the error in the estimated Earth-fixed velocity. The main contributors are body-fixed velocity error, and heading error. The error in estimated body-fixed velocity is mainly determined by the low-frequency errors of the DVL itself, or errors attributed to uncertainty in the installation calibration of the DVL. These errors are not observable if the vehicle is traveling along a straight line and without position aiding. High frequency velocity errors are on the other hand estimated by means of the IMU. As for the error in heading, it is determined by the gyro-compassing capability of the integrated system.

The main focus of this paper is the testing and integration of the new Nortek 500 kHz DVL onboard the Kongsberg Maritime (KM) HUGIN AUV platform. The HUGIN AUV has a state-of-the-art proprietary INS which can run both in-situ and in post-processing. As described in subsequent sections, the KM INS with a suitable velocity aiding sensor (in this case the Nortek DVL) satisfies all the navigation prerequisites mentioned above. The reminder of this paper is organized as follows: Section II describes the HUGIN AUV and relevant subsystems; succeeded by a presentation of the Nortek DVL in Section III. The experimental setup is described in Section IV, followed by sea trial results in Section V. Some concluding remarks are given in Section VI.

II. HUGIN AUV

In this work, the platform used for testing the Nortek DVL is the KM HUGIN AUV. A picture of the vehicle is shown in Fig. 1. It is a high-end, medium-size AUV with space and energy necessary to host the KM HISAS 1030/1032 interferometric synthetic aperture sonar (SAS), as well as multiple other payload and navigation sensors. HUGIN has a maximum depth rating of 6000 m, and it is capable of carrying out a wide range of applications (without making compromises) including, but not limited to, mine-counter measure (MCM), maritime security operations, and hydrographic surveying.

The navigation performance of all the KM AUVs is of great importance, and it is under continuous development by KM and the Norwegian Defence Research Establishment (FFI). The design philosophy has been to employ the best possible IMU, together with a large toolbox of aiding sensors and techniques [2], [4], [7], [8], [11]. The KM INS found in the AUV in-situ, is named NavP. Similarly, the KM INS implemented for simulation and navigation post-processing is named NavLab [12]. The mathematical foundation of the two is close to identical, with NavLab having some additional functionality. NavLab has been used extensively for post-processing of HUGIN navigation data since the late 1990s, and later with both the KM Munin AUV and the Hydroid REMUS AUVs. NavLab is the main analysis tool in this work. Further details on the KM INS are provided in Section II-A and II-B.

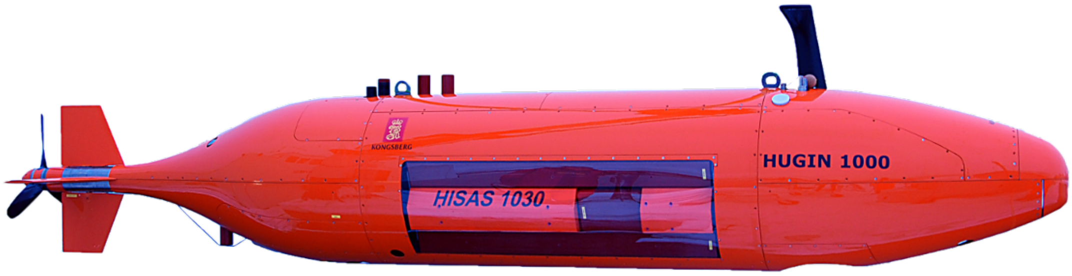

Fig. 1. The Kongsberg Maritime HUGIN AUV. The DVL it mounted in the lower part of the aft section, right in between the visible downward transducer and the HISAS window.

A. Navigation System Overview

An INS calculates position, velocity and attitude using high frequency data from an IMU which typically consists of three accelerometers measuring specific force and three gyros measuring angular rate, all relative to the inertial space. Due to inherent errors in the gyros and accelerometers, the solution of the navigation equations embedded in the INS will have an unbounded drift unless counteracted. An aiding framework is consequently required to bind the error growth.

In order to fuse the data from the INS and the aiding sensors some form of filtering must be implemented. This is typically accomplished using an error-state Kalman filter (KF), which is also the case in NavP and NavLab. The data fusion provides a much higher total navigation performance than obtained by the independent navigation sensors alone. A simplified schematic diagram showing the integration of DVL data is shown in Fig. 2, where the KF input is the difference between the output from the appropriate aiding sensors and the INS. The output from the KF includes estimates of the slowly varying systematic (colored) errors of the navigation sensors, as well as sea current when using DVL water-track data [8].

In Section V-B, the NavLab-estimated systematic errors of the Nortek DVL bottom-track data are used for assessing the long term sensor performance. Similarly one can also use the estimated systematic error and known total velocity error (since a very good NavLab reference solution exists) to obtain an estimate of the random (white) noise level. The latter is done in Section V-C, where a comparison is done between the NavLab-estimated white noise, and the corresponding figure of merit (FOM) estimate provided by the Nortek DVL itself.

B. Navigation Post-Processing

When discussing navigation one must distinguish between performance in-situ and in post-processing. The NavP navigation performance obviously determines where the vehicle actually collects its data and is therefore important for mission success. Depending on the application however, it may be desirable to enhance the navigation accuracy and integrity further in post-processing.

The enhanced accuracy when using NavLab with the KM AUVs is due to carrying out smoothing, which is a stochastic estimation technique that utilizes both past and future measurements [13]. Smoothing is especially effective when position updates are sparse. The smoothed solution from NavLab is also beneficial when doing sensor calibration, e.g. for finding DVL misalignment and scale factor coefficients. As described in Section IV, part of the experiments in this work involved integration calibration of the Nortek DVL, and part involved performance evaluation. The data produced by NavLab were used for both purposes.

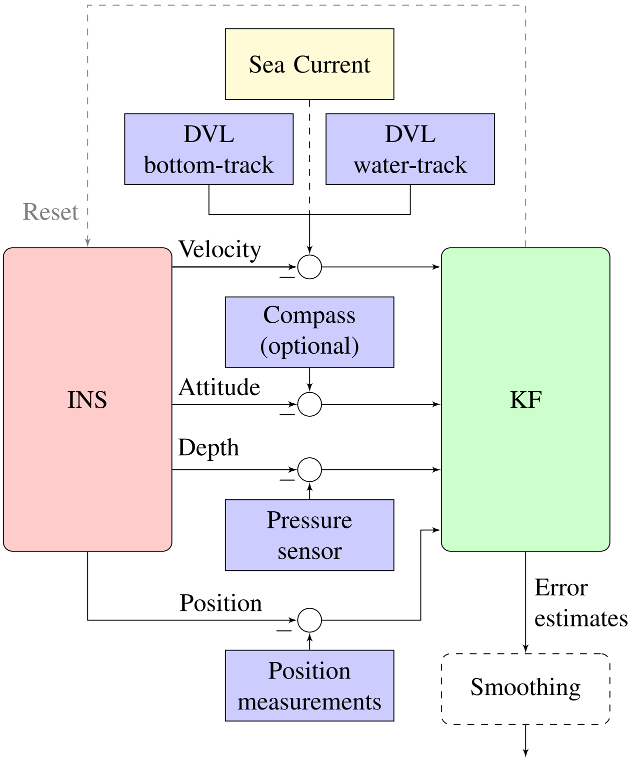

Fig. 2. Simplified structure of the KM INS with error state KF. A wide range of position aiding tools can be integrated, as well as additional velocity aiding sources (not shown in the figure). The main focus in this paper is on DVL bottom-track.

III. NORTEK DVL

The DVL used in these sea trials is the newly developed Nortek 500 kHz DVL. Built around a modern digital hardware platform, it comprises advanced signal processing capabilities, including Nortek's proprietary bottom detection algorithm [14]. This offers improved detection at low SNR allowing

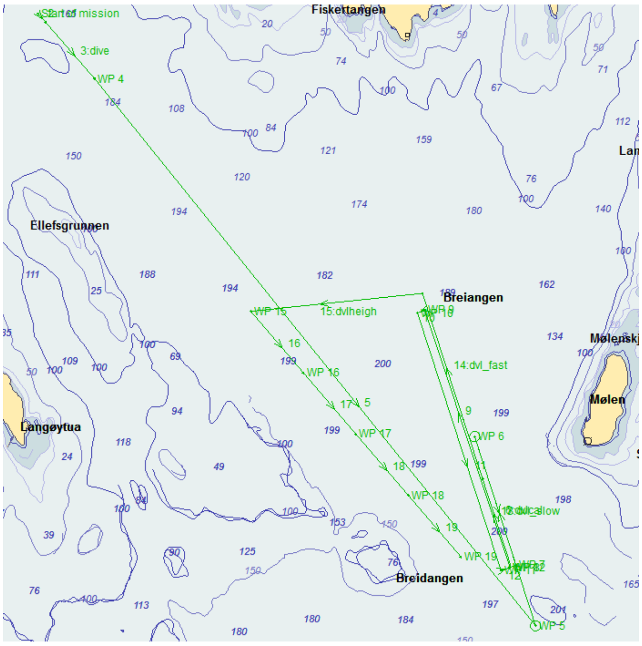

Fig. 3. Mission plan for DVL data collection. The water depth in the entire test area is close to 200 m. The longest straight line seen in the mission plan is close to 7 km.

bottom-tracking at greater range. It also greatly enhances bottom discrimination which improves detection in difficult conditions and allows reliable detection at very close range.

Another aspect of the new detection method is the ability to estimate Doppler measurement noise. Nortek's data format includes a FOM with every velocity estimate (per ping, per beam, per velocity). The FOM provides an estimate of the random velocity uncertainty; a measure of the instantaneous Doppler white noise standard deviation. This is useful for quality assessment of the velocity data and it may be used as measurement noise input to the KF in an INS. The KF uses estimates of each aiding sensor's standard deviation to distribute relative weight between the various sensors' data. If the KF is set up with a low standard deviation for the DVL, then DVL velocities are given much weight in the navigation solution. If the standard deviation is set high, then relatively less weight is placed on the DVL data. Traditionally, these standard deviation estimates are constants based on previously collected data and remain fixed throughout a mission. In post-processing of navigation data however, it is customary to actively change these settings to improve the final navigation solution. An experienced surveyor may observe that the DVL velocity data become noisy in a certain part of the mission, and as a result may increase the tuning noise level for that part of the track in order to avoid accumulating large errors in the KF. In reality, the DVL's velocity noise level varies continuously with the internal SNR (signal strength varying with range and bottom conditions) and interference (multipath echoes or external signals). A step toward optimal navigation accuracy would thus be achieved if the KF could change the standard deviation settings dynamically in accordance with the sensors' actual instantaneous noise levels. Now, with the Nortek DVL's FOM this may be done in real time. This would even be possible for a KF working directly with beam velocities rather than xyz-velocities. Section V-C in this paper will be devoted to the verification of the FOM as a valid estimate of the instantaneous Doppler white noise level.

As mentioned, another critical aspect of INS aiding is the time synchronization between the various sensors. In an environment with dynamics, a comparison of DVL and INS velocities is only valuable if taken at the same instance in time. What the DVL measures is an average velocity over the time the acoustic pulse is in the water. This averaging represents a time lag in the DVL velocity compared to the INS velocity. Further lag may result from the time the DVL uses to locate the true bottom echo and process this to derive the Doppler velocity. The Nortek DVL solves the time synchronization problem by providing embedded time stamps with 1 ms precision with every measurement (per ping, per beam, per velocity). These time stamps relate the time the velocity was valid to the absolute time the trigger pulse was received and to the time the data packet was transmitted. Further improvement of the time synchronization may be achieved when connecting to the Nortek DVL through its Ethernet connection and applying the Precision Time Protocol (PTP, IEEE 1588). This will allow clock synchronization of the DVL and the INS on the order of microsecond precision.

IV. EXPERIMENTAL SETUP

This section describes the experimental setup, employed navigation sensors, mission trajectory, and the post-processing of the raw navigation data. The experimental results are discussed in Section V.

A. *Mission Description*

The data in this paper were collected in June 2016 in the Oslo fjord, outside the city of Horten, Norway. The mission plan is shown in Fig. 3. The mission plan was divided into two parts: (1) one hour straight-line navigation performance evaluation at 30m altitude and 2 m/s fixed forward speed; and (2) about two and a half hours with general performance assessment and calibration at various speeds and AUV altitudes 4-50 m. The water depth in the entire test area is close to 200 m. Prior to conducting the actual mission, the AUV also collected data for one hour while on deck (used for INS heading initialization). During the mission the AUV was followed by a surface vessel the entire time in order to log GNSS-USBL (ultra-short baseline) measurements at about 1Hz. A RTK GPS system and a KM HiPAP 502 on board the vessel were used for this purpose.

B. *Sensors*

The following is a list of navigation sensors on the HUGIN of relevance for the work in this paper (uncertainties are 1σ):

- Honeywell HG9900 IMU
  0.002°/h gyro bias, $25\mu g$ accelerometer bias
- Paroscientific pressure sensor
  0.01% of full-scale (3000 m)

Fig. 4. Underneath HUGIN; the Nortek 500 kHz DVL to the left and the TRDI WHN 300 kHz DVL to the right.

- TRDI WHN 300kHz DVL
  ±0.4% ±0.2 cm/s, 200 m range
- Nortek 500kHz DVL
  ±0.2% ±0.1 cm/s, 180 m range (200 m next release)
- SAIV SD208 CTD (sound speed scaling of DVLs)
- GNSS-USBL
  RTK GPS
  HiPAP 502: $<$ 2 cm in range, 0.1° directional

The DVLs were both synchronized to an external trigger, running at 2Hz. No noticeable acoustic interference was seen in the investigated data. No other acoustic sensors close to the respective DVL frequencies ran during the mission. Electrically all the sensors on HUGIN are isolated. The DVLs mounted on the HUGIN AUV are shown in Fig. 4.

A common and distributed clock and time stamp system is available onboard HUGIN, making sure that all the payload and navigation measurements are time synchronized. While it has been successfully done in HUGIN, time stamping of DVL data is generally not straight forward as it depends on internal DVL modes and altitude among other things. However, as described in Section III, the Nortek DVL supports network time synchronization and embedded time stamping, hence making the integration easier.

C. Data Processing

At the time the test was carried out only 1PPS (pulse-per-second) time synchronization regime was available on the HUGIN side, hence a simple time offset calibration between the HUGIN clock and the Nortek internal clock was done both pre-mission (in order to obtain a nominal offset value) and in-mission (continuously for updating the nominal offset). The calculated time varying offset, together with the Nortek time stamp, provided an accurate DVL data time stamp (relative to the HUGIN time) when doing navigation post-processing.

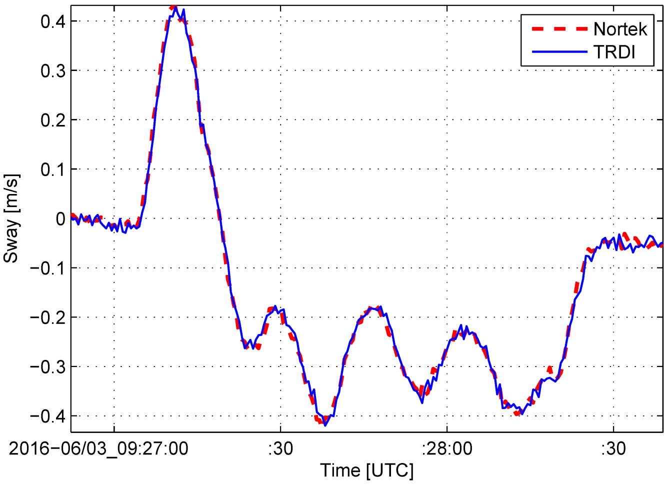

Fig. 5. Comparison of Nortek and TRDI velocity (only the sway component shown); both sensors' data are lever arm compensated to the IMU location, and with misalignment and scale factor calibration applied. The data from the two DVLs show good agreement, including time stamping.

As mentioned in Section II-B, NavLab-processed data were used in this work for analyzing the performance of the Nortek DVL; both when looking at systematic (colored) and random (white) errors, as well as the (related) dead-reckoning performance. The reference solution used for assessing the performance, and for doing the Nortek DVL calibration (misalignment and scale factor), was based on doing a full INS post-processing in NavLab, also utilizing the high-precision and continuous GNSS-USBL data for aiding.

Finally note that the two parts of data described in Section IV-A did not overlap in the post-processing, that is, the data from part (1) was not used for doing the DVL calibration. Only data from part (2) was used for this.

V. SEA TRIAL RESULTS

The TRDI WHN 300 kHz DVL has been part of the HUGIN navigation system for more than a decade, and the sensor is well integrated. It was therefore interesting to compare the velocity measurements of the Nortek DVL to those from the TRDI DVL. An example is shown in Fig. 5 for the sway velocity component during some dynamic turning maneuvers. The figure shows excellent agreement between the two DVLs, with instantaneous velocity differences on the order of mm/s, and time averaged much less. The agreement is also good on the time line. For the TRDI the time stamps were calculated externally by combining the time of the pulse going out and the acoustic time of flight of the pulse based on reported range to the bottom, and then adding a known latency. For the Nortek, the provided embedded time stamps were used.

A. Calibration

In order to achieve high accuracy with DVL aided INS, calibration of the system is essential; something KM has established accurate algorithms for doing. First the DVL data must be lever arm compensated to account for the difference

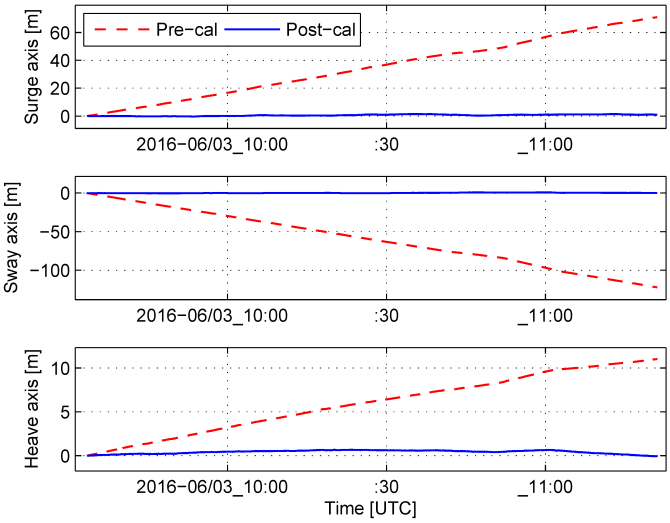

Fig. 6. Nortek DVL calibration; cumulative error growth along three axes versus time before and after misalignment and scale factor compensation.

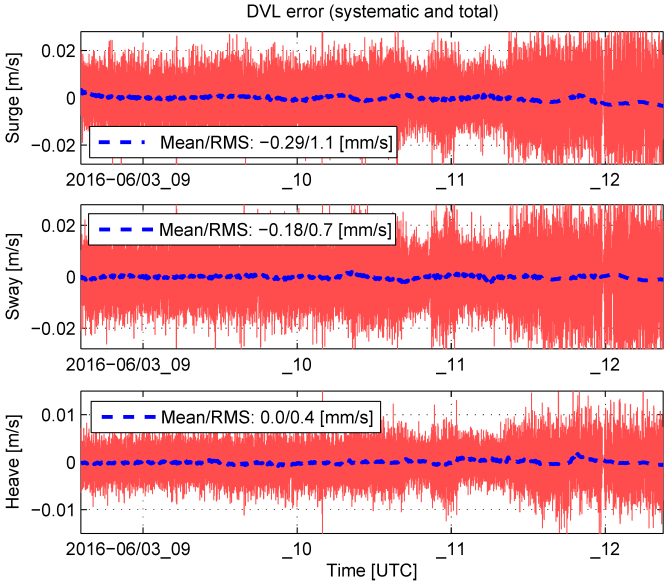

Fig. 7. NavLab-estimated total and systematic (colored) error of the Nortek DVL during the entire duration of the mission. The first hour of the plot was the straight-line navigation test, and none of this data was used for the DVL calibration part. The systematic error seen in blue (dashed) is indicative to the long-term accuracy. It should be noted that it may vary slightly from mission to mission, but the numbers are well within the specification of 0.2% of speed.

in physical position between the reference point in the DVL and INS. Second, the rotational mounting misalignment between the DVL and INS must be compensated for. Third, scale factor errors must be compensated for. Fig. 6 shows how errors accumulate with time (proportional to velocity error) without the above mentioned calibration steps. With an appropriate procedure carried out, the static errors will be close to removed. It should be noted that the alignment and scaling calibration absorbes any internal misalignment of transducer beam angles as well as any bias in the sound speed used to scale the DVL velocities.

B. *DVL Performance*

After calibration and removal of all static DVL errors, a high-end DVL aided INS should be capable of accurate navigation over long distances without external position aiding. The performance of the Nortek DVL itself, and the KM INS aided with the Nortek DVL were analyzed using NavLab. The high fidelity of the analysis was made possible due to the integration of continuous and accurate GNSS-USBL position measurements, as well as carrying out stochastic smoothing as outlined in Section II-B.

It may be instructive to separate the total DVL errors into three contributions: static errors, slowly varying systematic (colored) errors, and random noise (white). The static errors are effectively minimized using a calibration procedure as illustrated in Fig. 6, and after calibration the total errors should ideally be close to zero-mean. The total Nortek velocity error from each ping was estimated by subtracting the NavLab reference velocity from the DVL velocities. The red lines in Fig. 7 show the total DVL errors, which was found to be close to zero-mean along all three velocity axes.

The white measurement noise, which is further discussed for the Nortek DVL in Section V-C, yields a random walk type error that scales with the velocity standard deviation and the square root of time. The standard deviation of the white noise is typically used for describing the short-term DVL accuracy. As for the systematic error it is even more harmful to the overall navigation performance since it yields a position error drift which grows proportionally with time. Having a low systematic error is therefore important. If isolated, the slowly varying systematic error (being correlated from ping to ping) gives a measure of the long-term accuracy of the DVL, where both the mean and variation about the mean play a role.

As mentioned in Section II-B, NavLab applies a process model in the KF in order to estimate the systematic errors of the DVL (including what remains of the static errors). The white noise is modeled separately. The estimated systematic error of the Nortek DVL in the mission of discussion is seen as the blue (dashed) lines in Fig. 7. Mean and RMS values for the same lines and time span are also shown in the figure. The largest is in surge, where the systematic error RMS value was found to be 1.1 mm/s, corresponding to about 0.08% and 0.06% at the lowest and mean speed during the mission, 1.5 m/s and 2.0 m/s, respectively. The maximum speed was 2.85 m/s. Looking at the mean values alone the numbers are even lower. The Nortek DVL long-term accuracy thus appears to be well within the 0.2% of speed specification.

As a second performance test, a validation was also done to see how the Nortek DVL would perform in a traditional dead-reckoning (DR) computation, that is, by doing a direct numerical integration of the DVL velocities (after applying the standard calibration corrections and lever arm compensation) from a given initial position, and by using a presumed known orientation history. The following expression was evaluated:

$$\boldsymbol{p}(t) = \boldsymbol{p}(0) + \int_{t_0}^{t_f} \boldsymbol{R}^{\mathrm{L}}_{\mathrm{B}}(t)[\alpha \boldsymbol{R}^{\mathrm{B}}_{\mathrm{D}} \boldsymbol{v}^{\mathrm{D}}(t) - \boldsymbol{\omega}^{B}(t) \times \boldsymbol{r}^{B}_{\mathrm{BD}}]\, dt, \quad (1)$$

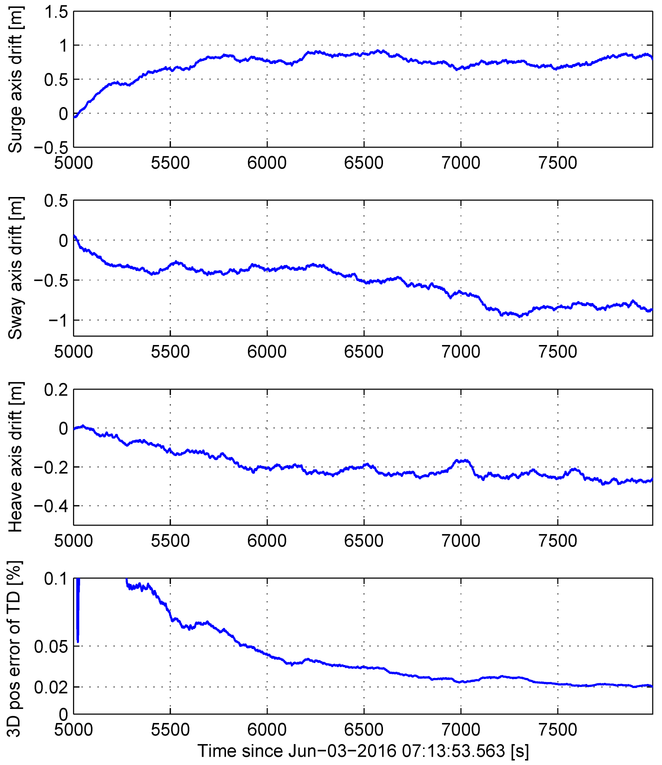

Fig. 8. Nortek DVL DR verification. The position error-drift of the numerical DR integration compared to the reference solution is visualized with drift along each body axis, as well of percentage of travelled distance. The AUV travelled about 6000 m during the 3000 s used for evaluating the performance. The position error at the end was 0.02% (DRMS) of TD. The specification of the KM INS when aided with DVL is typically 0.1% (DRMS) of TD, straight-line. The results from a DVL aided INS during autonomous missions are expected to vary somewhat from mission to mission due to heading alignment, but the performance of the Nortek DVL illustrates that it is a viable alternative for high-end underwater navigation.

where the terms $\alpha \boldsymbol{R}^{\mathrm{B}}_{\mathrm{D}}$ and $\boldsymbol{r}^{B}_{\mathrm{BD}}$ are the Nortek DVL calibration parameters and lever arm, $\boldsymbol{v}^{\mathrm{D}}(t)$ is the raw Nortek DVL velocity, and $\boldsymbol{p}(0)$, $\boldsymbol{R}^{\mathrm{L}}_{\mathrm{B}}(t)$, and $\boldsymbol{\omega}^{B}(t)$ are the initial position, orientation and angular rate from the reference solution. The reference solution was the smoothed NavLab output with all the GNSS-USBL and TRDI measurements utilized for aiding. Consequently, the test effectively evaluates the magnitude of the total errors of the Nortek DVL velocities. Some contribution must also be attributed to the heading uncertainty of the reference solution which was 0.03° in the available dataset. Nonetheless, it provides a good indication of the dead-reckoning ability of the Nortek DVL. The convincing results of the straight-line navigation test are shown in Fig. 8. The position error drift in this particular test was found to be 0.02% (DRMS) of travelled-distance (TD). While the results will vary from mission to mission due to e.g. varying heading alignment, it indicates that using the Nortek DVL with the KM INS should be able to meet the typical DVL aided KM INS specification of 0.1% (DRMS) of TD straight-line. The results are also in good agreement with the results in Fig. 7.

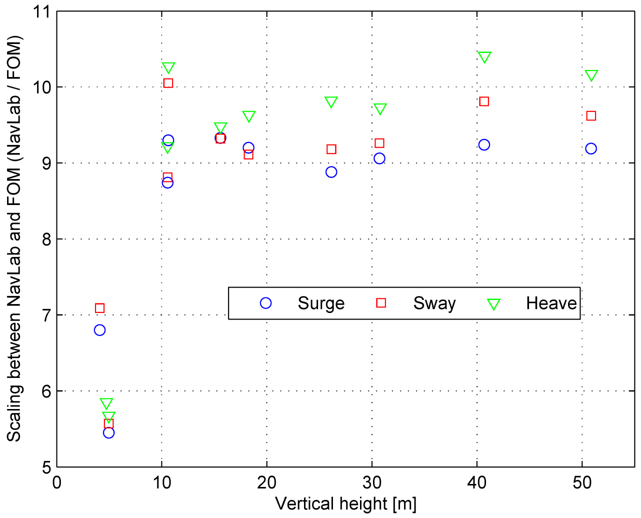

Fig. 9. Scaling of Nortek FOM to make it fit the NavLab-estimated white noise in the surge, sway and heave axes.

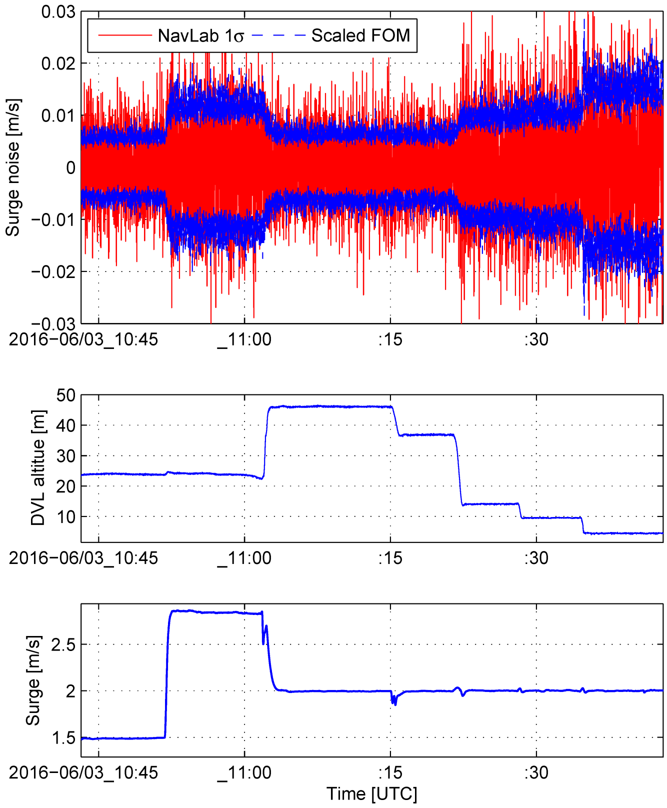

Fig. 10. Nortek FOM (using the numbers in Fig. 9 for scaling) versus NavLab-estimated white noise in surge. The sway and heave axes look similar in terms of matching the Nortek FOM with the NavLab estimate. The scaled FOM gives a good estimate of the instantaneous white noise level. As easily seen from the FOM, the noise level changes with altitude and speed.

C. FOM and White Noise

By subtracting the systematic error from the total error one can obtain an estimate of the white noise. It has been an objective for Nortek to provide a FOM representative of the white noise level. In the firmware used in this test, the FOM had been implemented without proper scaling. The first step was thus to establish a scaling factor between the FOM and the white noise level estimated by NavLab. Fig. 9 shows this scaling factor as a function of the vertical distance to the bottom. As expected, the scaling factors for the different axes are comparable, and close to constant for vertical range more than 10 m. Below 10 m the factors are seen to drop. More data will be required to find the accurate scaling at low altitudes, but for now a linear scaling will be assumed below 10 m.

By applying the scaling in Fig. 9 to the FOM data from the Nortek DVL, the resulting FOM may be compared to the white noise level from NavLab. Fig. 10 shows how the white noise level varies during the mission as a function of altitude and speed (red). The figure also shows the scaled FOM (blue) which is seen to accurately follow the white noise level estimated by NavLab. This shows that, for the first time, a DVL has the ability to estimate its own noise level in-situ.

VI. CONCLUSION

The DVL and IMU are the key AUV navigation sensors, enabling submerged operation for long periods of time. To utilize the velocity accuracy offered by the DVL, mounting misalignment between the IMU and the DVL must be minimized, sound speed must be accurately calculated and the sensor data properly time stamped.

The sea trial with the KM HUGIN AUV indicates that the performance of the Nortek 500 kHz DVL is well within specification, making it a viable alternative to state-of-the-art AUV navigation. The Nortek DVL also possesses some useful features like embedded time stamping and FOM data, which both make the system integration easier. It has been shown that the Nortek FOM provides an accurate real-time measure of Doppler white noise level.

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